

# Overview of CAT: A Market Design Competition

Version 1.1  
June 6, 2007

## 1 Introduction

This document presents detailed information on the Trading Agent Competition (TAC) Mechanism Design Tournament, also known as the CAT Tournament, being held in July 2007. CAT is an abbreviation of *catalactics*, the science of economic exchange. The document has been prepared by the following people: Enrico Gerding (University of Southampton, UK), Peter McBurney (University of Liverpool, UK), Jinzhong Niu (Brooklyn College, USA), Simon Parsons (Brooklyn College, USA) and Steve Phelps (University of Liverpool and Victria Ltd, UK). Responsibility for the contents of the document rests with Peter McBurney, GameMaster for the TAC Mechanism Design Tournament in 2007. The document is version 1.1 of *catoverview.pdf*, and is issued on 6 June 2007. The document should be cited as [4].

The 2007 CAT Tournament is hosted by the Trading Agents Competition, which will be held at AAAI 2007, in Vancouver, BC, Canada. The document needs to be read in conjunction with the other documents and software available for public download on the sourceforge site for project *jcat*, the CAT Market Design Competition:

<http://sourceforge.net/projects/jcat/>

The CAT tournament software has been developed by a team at Brooklyn College, comprising Kai Cai, Rayman Chan, Jinzhong Niu and Simon Parsons. The resulting software is a modification of the Java Auction Simulator API (JASA), initially developed by Steve Phelps. Further information about JASA is available from here:

<http://www.csc.liv.ac.uk/~sphelps/jasa/>

Operational testing of the CAT tournament software, simulation analysis needed to decide tournament parameters, and development of web interfaces, has been undertaken by Arthur Coussy (Ecole des Mines de Nantes, France, and University of Liverpool, UK), Enrico Gerding, Mathieu Lassalle (Ecole des Mines de Nantes, France, and University of Liverpool, UK), Peter McBurney, Thierry Moyaux (University of Liverpool, UK), Jinzhong Niu and Dave Shield (University of Liverpool, UK).

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## 1.1 Change Log

- Version 1.0: First version.
- Version 1.1: Revisions:
  1. Section 1 — Minor edits and additions to text.
  2. Section 2.4.1 — Details on client-server operation of game.
  3. Section 2.4.3 — Price lists broadcasted to all specialists.
  4. Section 2.4.4 — A specialist's information about the number of traders.
  5. Section 3.2 — Information added on the assessment score algorithms.
  6. Section 5 — Information added concerning the Qualification Stage and Final Stage of the TAC 2007 CAT Mechanism Design Tournament.

## 2 Game Overview

The game consists of *trading agents* (or simply *traders*), i.e., buyers and sellers and *specialists*. Each specialist operates and sets the rules for a single exchange market, and traders buy and sell goods in one of the available markets. In the CAT competition the trading agents are provided by the CAT game, whereas specialists (and the rules of the markets) are designed by the entrants. Each entrant is limited to operate a single market.

A CAT game consists of a CATP server and several CATP clients, which may be trading agents or specialists (markets). CATP clients do not talk to each other directly; instead they connect to the CATP server through socket and the server responds to messages from clients and forward information if needed.

### 2.1 Trading Agents

Each trading agent is endowed with a *trading strategy* and a *market selection strategy* and assigned private values for the goods being traded. The traded goods are assumed to be homogenous and non-divisible. Furthermore, each buyer and seller has a certain

demand and supply. Private values and the demand and supply of the markets are allocated by the CATP server, and are reset at the start of each day. Although private values remain constant during a day, these values may also change from day to day.

The trading strategy is used to generate bids and asks (also called shouts), whereas the market selection strategy is used to select a market or *specialist*. More detailed information about the trading agents' strategies is provided in Section 4.

Each trader is furthermore endowed with a limited budget that they can spend within a trading day. This budget prevents a trader from paying excessively high fees. Budget sizes are unknown to the specialist.

## 2.2 Specialists

Specialists facilitate trade by matching bids and asks and determining the trading price in an exchange market. Each specialist operates its own exchange market. Each entrant in the game is required to design a single specialist or market, which is achieved by implementing the following policies:

- *Charging Policy*. This policy sets the fees (also referred to as *price list* in this document) which are charged to traders and other specialists who wish to use the services provided by the specialist. Each specialist is free to set the level of the charges (from zero up). These are the following:
  - *Registration fees*. Fees charged for registering with a specialist.
  - *Information fees*. Fees for receiving market information from a specialist.
  - *Shout fees*. Fees for successfully placing bids and asks.
  - *Transaction fees*. A flat charge for each successful transaction.
  - *Profit fees*. A share of the profit made by traders, where a trader's profit is calculated as the difference between the shout and transaction price.

The first 4 types of fees are each a flat charge, and the last one is a percentage charged on the profit made by a trader. A trader pays the registration and information fees at most once every trading day. An example of how the last three fees are applied is given in Section 2.3.

- *Accepting Policy*. This policy determines which shouts (i.e., bids and asks) are accepted. A specialist has the option to reject shouts which do not conform to the specialist's policy. For example, a "beat the quote" policy requires a shout to beat the market quote. If the received shout violates this, it can be immediately rejected and will not be considered for a transaction, allowing the trader to submit a new shout.
- *Clearing Policy*. This policy determines the way in which bids and asks are matched. For example, in the well-known continuous double auction, the market is cleared as soon as new shouts arrive.
- *Pricing Policy*. This policy determines the transaction price of a matched bid and ask. The most common is to set the price half way between the bid and ask.

Specialists are permitted and even encouraged to have adaptive strategies such that the policies change during the course of a game in response to market conditions.

A specialist will compete with the other specialists in the market on two levels. First of all, specialists compete for traders; at the beginning of each day of the game, a trader will choose a single specialist to register with and trade for that day (more details are provided below). A specialist therefore has to design the market rules and fees such that the specialist attracts both buyers and sellers. Secondly, specialists compete to obtain the highest score in the tournament. This score each day is determined not only by the profit made by the specialist that day, but also by the market share (the proportion of traders registered with the specialist compared with other specialists), and the proportion of bids and asks which resulted in a successful transaction with that specialist on that day. These factors together are aggregated to determine the winner of the competition. A detailed description of the scoring rules are provided in Section 3.

### 2.3 Transaction Example

We present a simple example to illustrate the principles involved in a specialist charging fees to traders. We ignore the *registration* and *information* fees in this example and consider interaction between a single buyer, seller, and specialist.

Suppose a specialist has the following charging policy:

- Shout fees = \$2
- Transaction fees = \$5
- Profit fees = 10%

The *buyer* has private value of \$90 for each unit of the good and the *seller* has a private value of \$80; these values are unknown to the specialist. Suppose that traders shout their true values: thus, the buyer places a bid of \$90 and the seller submits an ask of \$80, and the specialist sets the transaction price to be \$86.

The fees are calculated as follows. The transaction and shout fees amount to \$7 for each trader. A trader's *profit* is calculated as the difference between the shout and the transaction price. In this case the difference between the ask and the transaction price is  $\$90 - \$86 = 4$ , and the profit fees for the buyer are  $\$4 \times 0.10 = \$0.40$ . The seller's profit is equal to  $\$86 - \$80 = \$6$  and the seller pays a profit fee of  $\$6 \times 0.10 = \$0.60$ . The total fees for the buyer and seller are \$7.40 and \$7.60 respectively.

### 2.4 Game Procedure

In this Section we provide an detailed overview of the course of a CAT game. Each game consists of several *trading days*<sup>1</sup>, and each day consists of several rounds. We now explain what occurs at various stages in the game.

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<sup>1</sup>Note that a trading day is a virtual day and is different from an actual tournament day in real time.

#### 2.4.1 Before the Game Starts

The TAC 2007 CAT Mechanism Design Tournament will be operated from a server machine hosted in the Department of Computer Science at the University of Liverpool, UK. Entrants to the Tournament write programs to operate as specialists in the game. Participation in the Tournament will require a specialist to operate on a client machine, hosted by the entrant concerned, which is able to maintain continuous or nearly-continuous connection to the Internet. Prior to commencement of the Qualifying Stage Game and of the Final Stage Game (see Section 5), entrants who have registered with the TAC site will receive instructions and a password to connect to the Liverpool server.

All traders and specialists are required to register, i.e., make their presence known to the server. Each trader and specialist will be assigned a unique ID at this point, and this ID will remain unchanged throughout the course of a game.<sup>2</sup>

After all specialists and traders have checked in (or until a time out), the server announces that the game is starting and informs the players of the number of rounds per day, and the length of each round (in milliseconds).

#### 2.4.2 At the Start of a Game

After the start of the game, but before the first day, the server sends a list of all traders and specialists in the game to all traders and specialists. In particular, at this point the specialist will learn the number of traders in the game, and the number of competing specialists. Note that the total number of traders remains constant for the duration of a game. Although the number of traders is known, the bidding and market selection strategies used by traders are not made public and these may change over time.

#### 2.4.3 Before a Day Starts

Before the start of *each day*, the specialists are required to announce their price lists. This information will be broadcasted to all traders and specialists. If a specialist fails to announce its fees before the actual day begins, the specialist will be excluded from trade for that day (but can participate again in the next day).

#### 2.4.4 During a Day

**Specialist Selection** Once a day has started, traders can register with one of the specialists (and only one specialist). Their choice of specialist is determined by their *market selection strategy* and depends on both the announced fees for that day, such as the registration fee, but also on the profits obtained in previous days. Different traders can have different market selection strategies (more details of the various market selection strategies in the game are given in Section 4.2), and the actual strategy used by a particular trader and the parameter settings remain hidden. The strategies used by the

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<sup>2</sup>In case a specialist loses connection with the server at any time during the game, it is the responsibility of the specialist to recognize this event and to reconnect again with the server. When reconnecting, the specialist will log-in again (using the ID given by the server) and re-enter the on-going competition starting with the next trading day.

traders may also change over time. Whatever their strategy, however, obviously traders tend to go to specialists where they expect the highest profits.

**A Trader’s Information about Specialists** In addition to registering, at any time during a trading day, traders can subscribe to receive information from one or more specialists. This information may not be free, and the price for a subscription (a fixed fee) is set by each specialist individually.

The information that traders receive from a specialist when subscribed is the following: (1) all of the successfully submitted shouts (i.e., bids and asks) with that specialist<sup>3</sup>, and (2) all successful transactions. Traders can use this information to adapt their *trading strategy* which is used to generate the bids and asks (a detailed description of the set of trading strategies used by bidders is given in Section 4). Note that traders can subscribe to any number of specialists, and this is independent of the registration.

**A Specialist’s Information about Other Specialists** As mentioned above, before the actual start of the day, specialists are automatically informed about the price lists set by all the specialists in the game. Alternatively, a specialist may freely request this information at any time once a day has started. Furthermore, at the end of each day, the specialists are automatically informed about both the profits obtained by all the specialists on that day (including their own profits) and the number of traders that have registered with each specialist.

In addition to the above free information, a specialist may subscribe to receive information about successful shouts and transactions from other specialists, just as traders can. Specialists who choose to do so have to pay the same fee as the traders. A specialist may use this information, for example, to track the decisions made by individual traders (such as their choice of specialist) using the unique trader ID which is provided with each shout. Also, the transaction success rate can be inferred from this information. Together with the information received at the end of the day regarding the profit and the number of traders, this can be used to calculate the score of other specialists (see section 3.2 for details on how this score is calculated).

**Trading** We now discuss the actual trading. A day is divided into a number of rounds, during which traders submit shouts to the specialist they are registered with. Each shout specifies the limit price for a *single unit* of the traded good.

A specialist has the option to either accept or reject a shout. This can be used to enforce a certain bidding rule. For example, the rule “beat the quote” requires a shout to beat the current market quote. A specialist is not required to follow any explicit protocol, however, and can choose to reject or accept any shout. Once a shout is accepted, it becomes *active* and remains active either until a transaction is successfully completed or when the trading day ends.

For each trader only a single shout (for a single quantity of the good) can be active at any time. This means that a trader has to wait until a transaction is completed (or until the next trading day) before placing another shout. The reason for this restriction

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<sup>3</sup>Shouts can be rejected by a specialist, in which case they are not passed on as information to other any traders. See below for details.

is to encourage early transactions and to avoid delays. Although a trader cannot submit a new shout when another shout is still active, the active shout can be revised by the trader by submitting a new shout. If the new shout is accepted by the specialist, this replaces the active shout. If the shout is not accepted, nothing changes. Note that traders do not pay any fees for revising their shouts. Also, note that revisions are not counted as new shouts and therefore do not directly affect the transaction success rate (see also section 3.2).

A specialist's task is to match bids and asks and to determine the transaction price. Matching bids and asks can be done at any time during a round, provided the transaction is valid. For example, a continuous clearing mechanism will try and match any incoming bids as soon as they arrive. Alternatively, clearing can occur at the beginning of each new round or only at the last round of the day. Whatever the timing, a transaction needs to be valid. This requires the bid price to match the ask price, and the transaction price to be set at some point in-between the bid and the ask price.

Note that active shouts persist across rounds, but expire as soon as a day ends.

### 3 Assessment

This section presents the process for assessment of entries to the CAT game. The core idea is that entrants will be assessed on multiple criteria (and not just the profits), which will be evaluated on a number of trading days.<sup>4</sup> The assessment criteria to be used are described in Section 3.2 below. In order to avoid effects arising from the fact that the tournament has a start-day and an end-day, not all the trading days will be used for assessment purposes. The process by which days will be selected is described in Section 3.1 below.

#### 3.1 Sampling Method

- **Step 1:** We choose a random starting day, and a random ending day. Both selections are made prior to the commencement of the game, and remain secret (from both game entrants and game organizers). The starting day will only be revealed at some point after it occurs. The ending day will only be revealed after completion of the game.
- **Step 2:** Prior to game commencement, we also randomly choose days between the starting day and the ending day, on which assessment will be undertaken. These days are called "Assessment Days". These selections also remain secret, and are made public one-by-one, at the end of each such selected day. In order to avoid manipulation, revelation that the assessment process has commenced may not be made until after the first several Assessment Days have occurred.

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<sup>4</sup>Multiple criteria have been used for this tournament in order to better reflect competition between real-world exchanges, where the need by an exchange to enable specialists (or their equivalents) to make profits must be balanced with the need to attract traders to the exchange to ensure liquidity.

- **Step 3:** At the end of each Assessment Day following revelation that the assessment process has commenced, the scores of each specialist against each criterion, and their total score, will be made public.

### 3.2 Scoring

Each specialist is assessed on three criteria on each Assessment Day, and these criteria are then combined into a single score for that day. These three criteria are as follows:

- **Profits:** The profit score of a specialist on a particular day is given by the total profits obtained by that specialist on that day as a proportion of the total profits obtained by all specialists on that same day. The profit score is thus a number between 0 and 1 (both inclusive) for each specialist for each day, and the profit scores for all specialists on the same day will sum to 1.
- **Market Share:** Of those traders who have registered with a specialist on a particular day, the market share score of a specialist on that day is the proportion of traders that have registered with that specialist on that day. Again, the market share score is a number between 0 and 1 (both inclusive) for each specialist for each day, and the market share scores for all specialists on the same day will sum to 1.
- **Transaction Success Rate:** The transaction success rate score for a specialist on a given day is the proportion of bids and asks placed with that specialist on that day which that specialist is able to match. Let us denote:
  - $N_b$  for the number of bids placed with a specialist on a given day,
  - $N_a$  for the number of asks placed with that same specialist on the same day, and
  - $N_m$  for the number of successful matches executed by that specialist from the bids and asks the specialist received that day.

Then, the Transaction Success Rate Score for that specialist on that day is calculated as:

$$\frac{2 \cdot N_m}{N_b + N_a}$$

In the case where both  $N_b$  and  $N_a$  are zero, the Transaction Success Rate score is calculated as zero. Thus, the Transaction Success Rate score for any specialist on any day is also a number between 0 and 1 (both inclusive). Note that *revised* shouts (see also section 2.4.4) are not counted as new bids or asks, and therefore do not affect the values of  $N_b$  and  $N_a$ .

Each of these three criteria results in a value for each specialist for each day between 0 and 1. The three criteria are then weighted *equally* (i.e., weighted one-third each) and added together to produce a combined score for each specialist for each Assessment

Day. Scores are then summed across all Assessment Days to produce a final game score for each specialist. The specialist with the highest final game score will be declared the winner of the tournament.

## 4 Trading Agent Strategies

This section describes the different strategies that are used by the trading agents. Each trader is equipped with both a bidding strategy for generating bids and asks, and a specialist selection strategy for selecting a specialists each day.

### 4.1 Bidding Strategies

Each trader uses one of the following four strategies: GD (Gjerstad Dickhaut), ZI (Zero Intelligence), ZIP (Zero Intelligence Plus), and RE (Roth and Erev). These strategies are selected since they have been extensively researched in the literature and some of these have shown to work well in practice. Note that the strategy used by a trader is private knowledge, and is not revealed to the specialists. Also, the proportion of traders executing each of the four strategies will not be revealed to specialists; this proportion is likely to change over time as the game proceeds. Finally, most of these strategies are adaptive and optimize their bidding behaviour.

We now explain the strategies in more detail.<sup>5</sup>

#### 4.1.1 Zero-Intelligence

One of the most prominent strategies that have been developed over the years is Gode and Sunder's *Zero-Intelligence* (ZI) trading strategy [6]. The ZI agent is not motivated to seek trading profits and ignores all market conditions when forming a bid or ask by selecting a bid or an ask drawn from a uniform distribution over a given range (hence the term zero-intelligence). In terms of our categorisation, ZI is non-history-based and non-reactive as it does not consider the market condition in its decision-making process.

Specifically, Gode and Sunder considered the performance of both constrained and unconstrained ZI traders, which they respectively term ZI-C and ZI-U traders. The former are subject to budget constraints and are not allowed to trade at loss. The latter, however, are allowed to enter loss-making transactions (i.e. they are allowed to submit a bid (ask) that is higher (lower) than their limit (cost) prices). In more detail, the ZI-C buyer draws a bid from a uniform distribution between the minimum allowed bid and its limit price. The ZI-C seller forms an ask from a value drawn from a uniform distribution between its cost price and  $p_{max}$ , beyond which we assume no transaction can take place. For the ZI-U traders, the shout price is drawn from a uniform distribution between 0 and  $p_{max}$ . In the CAT game, we only consider ZI-C traders.

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<sup>5</sup>The descriptions of the trading strategies are taken from [9] and [7] with permission of the authors.

### 4.1.2 Zero-Intelligence Plus

The *Zero-Intelligence Plus* (or ZIP) strategy was first designed by [1] to show that more than zero-intelligence is required to achieve efficiency close to that of markets with human traders. ZIP has subsequently been used in a number of works as a benchmark for strategy evaluation (e.g. [2, 8, 10]).

Specifically, while the ZI strategy ignores the state of the market and past experience, the ZIP strategy uses a history of market information and is of the predictive class, adapting the agent's profit margin to the future market conditions. That is, the agent increases or lowers its profit margin to remain competitive in the market. In this context, the profit margin determines the difference between the agent's limit price and the shout price.

At the beginning of the trading day, the agent has an arbitrarily low profit margin which it increases or decreases, depending on the different market events (submitted offers by buyers and sellers and any successful transactions) during the trading period. The ZIP buyer increases its profit margin whenever events in the market indicate that it could acquire a unit at a lower price than its current shout price, given by its profit margin. For a ZIP seller, if its last shout resulted in a transaction and its shout price was less than the transaction price, this indicates that it could transact at a lower price which would necessarily increase its profit margin.

Conversely, ZIP buyers and sellers reduce their profit margin when this margin is too high to remain competitive. In this case, the buyer would have market power (to influence the trend of ask prices) if the seller were to decrease its profit margin whenever an unsuccessful bid is submitted (since a series of unsuccessful bids would unnecessarily decrease the seller's margin). Similarly, the buyer should not lower its profit margin after each unsuccessful ask submitted in the market. Thus, a buyer lowers its profit margin only after a submitted bid is rejected, while a seller lowers its profit margin only after an unsuccessful ask. It is also necessary to consider the trader's strategy after a successful shout. If the last bid was successful, the unsuccessful seller lowers its profit margin so as not to be undercut by the competing sellers. The unsuccessful buyer lowers its profit margin after an ask was accepted by a competing buyer. We summarise the bidding behaviour of the ZIP trader in Figure 1, where  $p_i^b(t)$  and  $p_j^s(t)$  are the most profitable offer to buy or sell of ZIP buyer  $i$  and seller  $j$  respectively, at any time during the trading period and  $s(t)$  denotes the price of the most recent shout. In particular, we have 6 different rules that specify when to increase or decrease the profit margin.

In this case, the profit margin is modified using a simple adaptive mechanism based on the Widrow-Hoff algorithm [11]. This is a continuous-space learning mechanism that back-projects the error between the current value and some desired value onto that current value. At any given time  $t$ , the ZIP trader  $i$  calculates the shout-price,  $p_i(t)$ , given its limit price,  $\ell_i$ , and trader's profit margin,  $\mu_i(t)$ , according to the following equation:

$$p_i(t) = \ell_i(1 + \mu_i(t)) \quad (1)$$

The ZIP seller's margin is raised by increasing  $\mu_i(t)$  and is lowered by decreasing

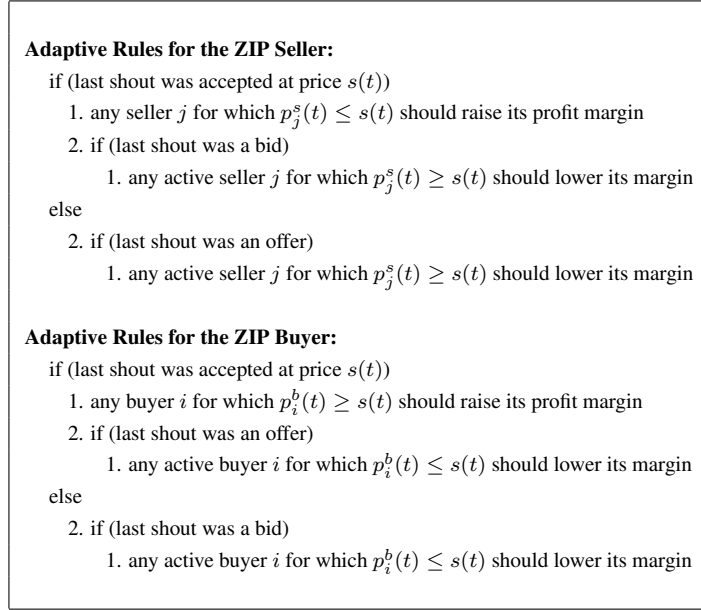


Figure 1: The ZIP Trading Strategy.

$\mu_i(t), \mu_i(t) \in [0, \infty)$ . Conversely, the ZIP buyer raises and lowers its profit margin by decreasing and increasing  $\mu_i(t)$  respectively. The initial profit margin,  $\mu_i(0)$ , is drawn from a uniform distribution over the range  $[0.1, 0.5]$  at the beginning of the simulation. The aim of dynamically modifying  $\mu_i(t)$  is for the trader's shout price to remain competitive against that of other participants. The learning mechanism of the profit margin is given in the following equations:

$$\begin{aligned}
\mu_i(t+1) &= (p_i(t) + \Gamma_i(t+1))/\ell_i - 1 \\
\Gamma_i(t+1) &= \gamma_i \Gamma_i(t) + (1 - \gamma_i) \Delta_i(t), \\
&\text{where } \Gamma_i(0) = 0 \forall i
\end{aligned} \tag{2}$$

$$\begin{aligned}
\Delta_i(t) &= \beta_i (\tau_i(t) - p_i(t)) \\
\tau_i(t) &= R_i(t) s(t) + A_i(t)
\end{aligned} \tag{3}$$

where the learning coefficient,  $\beta_i \in [0.1, 0.5]$ , determines the rate of convergence of the trader's shout price toward the target price  $\tau_i(t)$ . Here,  $R_i$  is a randomly generated coefficient that sets the target price *relative* to the submitted shout price  $q(t)$ , with  $R_i \in (1, 1.05]$  to increase  $\tau_i(t)$  (when increasing the profit margin) and  $R_i \in [0.95, 1)$  to lower  $\tau_i(t)$  (when reducing the profit margin).  $A_i(t)$  is an absolute perturbation, so the target price differs by at least a few units (of the minimum increment on the outstanding bid or ask), from even relatively small shout prices.  $A_i(t)$  is drawn from a

uniform distribution over  $[0, 0.05]$  for an absolute increase and over  $[-0.05, 0]$  for an absolute decrease. If we were to set  $\tau_i(t)$  to  $q(t)$ , the trader would never be able to submit an offer.

Furthermore, [1] improved their learning mechanism to minimise the effect of high-frequency changes in bids or asks, by considering the momentum (trend) of shout prices. The momentum-based updates are given in Equation 2. Specifically, the *momentum coefficient*,  $\gamma_i \in [0, 1]$ , determines the weight of previous shout prices on the change in the profit margin. When  $\gamma_i$  is equal to 0, the learning mechanism is myopic and ignores past quotes, while a high  $\gamma_i$  gives more weight to the trend of shout prices. In their simulations,  $\gamma_i$  is uniformly distributed over the range  $[0.2, 0.6]$ . Thus, the ZIP strategy has a set of 8 different parameters ( $\beta_i$ ,  $\gamma_i$ ,  $R_i$  and  $A_i$  to increase,  $R_i$  and  $A_i$  to decrease the margin,  $\mu_i(0)$  for the buyer and the seller) that determine how to increase or decrease the profit margin which is specified by the 6 different rules.

### 4.1.3 Gjerstad-Dickhaut

The GD strategy, developed by [5], is based on a belief function that an agent builds to indicate whether a particular shout is likely to be accepted in the market. In the GD strategy, buyers form beliefs that a bid will be accepted and similarly sellers form beliefs that an ask will be accepted in the market. The traders form their beliefs on the basis of the history of observed market data and, particularly, on the frequencies of submitted bids and asks and of accepted bids and asks resulting in a transaction. Given this information, the bidding strategy is to submit the shout that maximises the trader's own expected surplus, which is the product of its belief function and its risk-neutral<sup>6</sup> utility function. The GD strategy also implicitly considers the notion of recency, by limiting the trader's memory length,  $L$ , to a few transactions ( $L$  was set to 5 in Gjerstad and Dickhaut's simulations). The most recent history of shouts and transactions is considered.

In Gjerstad and Dickhaut's model, the seller's belief function,  $\hat{p}(a)$ , is based on the following assumptions. If an ask  $a' < a$  has been rejected, then an ask,  $a$ , will also be rejected. Similarly, if an ask  $a' > a$  has been accepted, then an ask submitted at  $a$  will also be accepted. Furthermore, if a bid  $b' > a$  is made, then an ask  $a' = b'$  would have been taken, since they assume that this ask  $a'$  would be acceptable to the buyer who bid  $b'$ . Similar assumptions are made about the buyer's belief function,  $\hat{q}(b)$ . We now define the bid and ask frequencies  $\forall d \in D$ , where  $D$  is the set of all permissible shout prices in the market, used in the belief function.

**Bid Frequencies:**  $\forall d \in D$ ,  $B(d)$  is the total number of bid offers made at price  $d$ ,  $TB(d)$  is the frequency of accepted bids at  $d$ , and  $RB(d)$  the frequency of rejected bids at  $d$ .

**Ask Frequencies:**  $\forall d \in D$ ,  $A(d)$  is the total number of ask offers made at price  $d$ ,  $TA(d)$  is the frequency of accepted asks at  $d$ , and  $RA(d)$  the frequency of rejected asks at  $d$ .

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<sup>6</sup>The risk-neutral agent uses a linear utility function and submits the price that maximises its expected profit.

The *Seller's Belief Function* for each potential ask price,  $a$ , is defined as:

$$\hat{p}(a) = \frac{\sum_{d \geq a} TA(d) + \sum_{d \geq a} B(d)}{\sum_{d \geq a} TA(d) + \sum_{d \geq a} B(d) + \sum_{d \leq a} RA(d)} \quad (4)$$

The *Buyer's Belief Function* for each potential bid price,  $b$ , is defined as:

$$\hat{q}(b) = \frac{\sum_{d \leq b} TB(d) + \sum_{d \leq b} A(d)}{\sum_{d \leq b} TB(d) + \sum_{d \leq b} A(d) + \sum_{d \geq b} RB(d)} \quad (5)$$

The seller's belief function is modified to satisfy the *NYSE spread reduction rule*. Thus, for any ask that is higher than the current outstanding ask, the belief function is set to 0 (i.e. that ask cannot be accepted). Similarly for the buyer, the belief that any bid submitted that is below the outstanding bid is accepted is 0 (i.e. that bid cannot be accepted).

Furthermore, because the belief function is defined over the set of all bids and asks within the trader's memory, the belief is extended to the space of all potential bids or asks allowed in the market, constrained by the outstanding bid and ask and the step-size of the belief function. Then, *cubic spline interpolation* is used on each successive pair of data items to calculate the belief of points in between the pair of data. In particular, a cubic function,  $p(a) = \alpha_3 a^3 + \alpha_2 a^2 + \alpha_1 a + \alpha_0$ , that ensures each two successive pair of points is constructed with the following properties:

1.  $p(a_k) = \hat{p}(a_k)$
2.  $p(a_{k+1}) = \hat{p}(a_{k+1})$
3.  $p'(a_k) = 0$
4.  $p'(a_{k+1}) = 0$

The coefficients,  $\alpha_i \forall i = \{0..3\}$ , that satisfy the above properties, are then given by the solution to the following equation:

$$\begin{bmatrix} a_k^3 & a_k^2 & a_k & 1 \\ a_{k+1}^3 & a_{k+1}^2 & a_{k+1} & 1 \\ 3a_k^2 & 2a_k & 1 & 0 \\ 3a_{k+1}^2 & 2a_{k+1} & 1 & 0 \end{bmatrix} \begin{bmatrix} \alpha_3 \\ \alpha_2 \\ \alpha_1 \\ \alpha_0 \end{bmatrix} = \begin{bmatrix} \hat{p}(a_k) \\ \hat{p}(a_{k+1}) \\ 0 \\ 0 \end{bmatrix} \quad (6)$$

Then, the buyer's belief function,  $q(b)$ , is constructed similarly using the pairs  $(b_k, \hat{q}(b_k))$  and  $(b_{k+1}, \hat{q}(b_{k+1}))$ . Having defined the belief functions, Gjerstad and Dickhaut proved that the beliefs are monotonically non-increasing (see [5] for further details). Thus, the belief of an ask,  $a > a'$  being accepted has to be lower than the belief of  $a'$  being accepted, and, similarly, the belief of a bid,  $b < b'$ , has to be lower than that of  $b$ . On this basis, we can clearly see that monotonicity of beliefs is an essential property that the belief function of the GD strategy must satisfy.

Given the belief function, the GD strategy forms an offer to buy or sell that maximises the trader's expected surplus (which is defined as the product of its belief function and its utility function,  $\pi(a)$ ). Because Gjerstad and Dickhaut consider risk-neutral traders, the utility function is linear and equals the profit of the traders; that is, the difference between the seller's ask price and its cost price, and the difference between the buyer's bid price and its limit price. When the trader's maximum expected surplus is

negative, there is no incentive to submit a bid or an ask and the trader abstains from bidding. The trader's utility function and its surplus maximisation is formulated as follows:

For a buyer  $i$ ,

$$\pi(b) = \begin{cases} \ell_i - b & \text{if } b < \ell_i \\ 0 & \text{if } b \geq \ell_i \end{cases}$$

For a seller  $j$ ,

$$\pi(a) = \begin{cases} a - c_j & \text{if } a > c_j \\ 0 & \text{if } a \leq c_j \end{cases}$$

$$b^* = \arg \max_{b \in (\mathcal{O}_{ask}, \mathcal{O}_{bid})} [\pi(b) \cdot q(b)] \quad (7)$$

$$a^* = \arg \max_{a \in (\mathcal{O}_{ask}, \mathcal{O}_{bid})} [\pi(a) \cdot p(a)] \quad (8)$$

#### 4.1.4 Roth-Erev

The Roth-Erev algorithm is a strategy designed to mimic human game-playing behaviour in extensive form games [3]. The strategy relies on the immediate feedback from interacting with the mechanism, namely the surplus that the agent was able to obtain in the most recent round of trading. Hence, this strategy is general-purpose enough to be used with any auction-mechanism, even where the trader does not have access to market-data, for example, in repeated *sealed-bid* auctions.

Roth-Erev is one of a class of strategies in which an agent  $a_i$  chooses its markup over its valuation price  $v_i$  as follows:

For  $a_i$  a seller:  $\zeta(i, t) = v_i + RL_{\lambda_i}(t)RL_{\mu_i}$

For  $a_i$  a buyer:  $\zeta(i, t) = v_i - RL_{\lambda_i}(t)RL_{\mu_i}$

where  $RL_{\rho_i}(t)$  is a *reward signal* which represents the most recent profits of agent  $a_i$ :

$$RL_{\rho_i}(t) = \Gamma_t(a_i) - \Gamma_{t-1}(a_i) \quad (9)$$

The function  $RL_{\lambda_i} : \mathbb{N} \rightarrow \Theta_i$  represents the output of learning algorithm  $\lambda$  where  $\Theta_i = [0, RL_{k_i}) \subset \mathbb{N}$  is the set of possible outputs from  $\lambda$ .

Parameter name	Semantics
$RL_{\lambda_i}(t)$	A function specifying the output from a 1-armed bandit learning algorithm
$RL_{\mu_i}$	A scaling factor used to map learning outputs onto actual prices
$RL_{k_i}$	The number of possible outputs from $RL_{\lambda_i}$

Table 1: Reinforcement-learning parameters

For the Roth-Erev strategy, we set:

<i>Parameter name</i>	<i>Semantics</i>
$RE_{k_i}$	The number of possible outputs
$RE_{\rho_i}$	The recency parameter
$RE_{\eta_i}$	The experimentation parameter $\eta$
$RE_{s_i}$	The scaling parameter

Table 2: Parameters for the Roth-Erev learning algorithm

<i>State variable</i>	<i>Semantics</i>
$RE_i(t)$	The output of the learning algorithm at time $t$
$RE_p(\theta, a_i, t)$	The probability distribution over each possible action $\theta \in \Theta_i$
$RE_q(\theta, a_i, t)$	The <i>propensity</i> for each possible action $\theta \in \Theta_i$

Table 3: State variables for the Roth-Erev learning algorithm

$$RL_{\lambda_i}(t) = RE_i(t) = \delta_i t \quad (10)$$

where  $\delta_t \in \Theta_i$  is a discrete random variable distributed as:

$$P(\delta_i t = x) = RE_p(x, i, t) \quad (11)$$

The propensities are initialized based on the scaling parameter  $RE_{s_i}$ ;  $\forall a_i \in A$  and  $\forall \theta \in \Theta_i$ :

$$RE_q(\theta, a_i, t_0) = \frac{RE_{s_i}}{RL_{k_i}}. \quad (12)$$

The  $RE_q$  are then updated based on the reward signal  $RL_{\rho}$ :

$$RE_q(\theta, a_i, t) = (1 - RE_{\rho_i})RE_q(\theta, a_i, t - 1) + RE_c(\theta, a_i, RL_{\rho_i}(t - 1)) \quad (13)$$

and then normalized to produce a vector of probabilities; let  $Q_{it}$  denote the sum of all the propensities for agent  $i$ :

$$Q_{it} = \sum_{\theta \in \Theta_i} RE_q(\theta, a_i, t) \quad (14)$$

Then,  $\forall \theta \in \Theta_i$  and  $\forall a_i \in A$ :

$$RE_p(\theta, a_i, t) = \frac{RE_q(\theta, a_i, t)}{Q_{it}} \quad (15)$$

## 4.2 Specialist Selection Strategy

Each trader also executes a strategy to select a specialist prior to each trading day. For some traders, on some trading days, this selection process is random. On other occasions or for other traders, selection by the trader of a specialist is based on the trader's prior transaction experience with the specialists in the market. Thus, the specialist selection strategy incorporates a learning component. The specific details of the learning component of specialist selection, and the occasions on which it is applied, is private information to the traders, and is not provided to specialists.

## 5 Qualification Stage

For the 2007 CAT Mechanism Design Tournament, a Qualification Stage will be organized prior to the final CAT Game. This Qualification Stage will be held on Wednesday 27 June and Thursday 28 June 2007. It is currently planned to run two simultaneous games during this period, with entrants allocated randomly to one qualifying game or to the other qualifying game. Thus, entrants in the Qualification Stage will not compete against all other entrants. Progression by a team from the Qualification Stage to the Final Stage will occur if the team's specialist participates in the qualification game with minimal functionality. In other words, if a specialist operates without problems in the Qualification Stage, even if that specialist scores poorly on the assessment criteria, then we expect it will be permitted to proceed to the Final Stage. However, performance in the Qualification Stage game may be used to provide individual seedings or handicaps in the Final Stage game.

The Final Stage of the TAC 2007 CAT Mechanism Design Tournament will be held with the TADA Workshop at AAAI 2007, in Vancouver, BC, Canada. The duration and dates of the Final Stage game will be determined in the light of the experience gained in running of the Qualification Stage, and will be announced by early July 2007.

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