

# TREE PATTERN MATCHING BY PUSHDOWN AUTOMATA

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FEBRUARY 5, 2009

# PATTERN MATCHING BY PUSHDOWN AUTOMATA

- 1 CREATE A SEARCHING NON-DETERMINISTIC PUSHDOWN AUTOMATON (SNPDA).
- 2 REMOVE  $\epsilon$ -TRANSITIONS (OPTIONAL).
- 3 DETERMINISATION OF SNPDA.

## REQUIREMENTS

- TREES MUST BE REPRESENTED IN PREFIX NOTATION.
- TREES MUST BE ROOTED, ORIENTED, RANKED AND ORDERED.

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# DETERMINISTIC PUSHDOWN AUTOMATON FOR PREFIX NOTATION

OF TREE WITH NODES HAVING RANK  $0, 1, 2, \dots, n$

$$(i + 1) S \mapsto a_i S^i$$

WHERE

$$i = 0, 1, 2, \dots, n$$



$$\delta(0, a_i, S) = (0, S^i)$$

$$S^0 = \varepsilon$$

# TYPES OF PATTERNS

- EXACT PATTERNS

$$P = a_{x_1} a_{x_2} \dots a_{x_n}$$

EXAMPLE:  $a_2 a_1 a_0 a_0$

- PATTERNS HAVING SUBTREES (TREE TEMPLATES)

$$P = a_{x_1} \dots a_{x_{k_1}} S^{p_1} a_{x_{k_1+1}} \dots a_{x_{k_m}} S^{p_m} a_{x_{k_m+1}} \dots a_{x_n}$$

EXAMPLE 1:  $a_2 a_1 S a_0$

EXAMPLE 2:  $a_3 S S a_2 S a_0$

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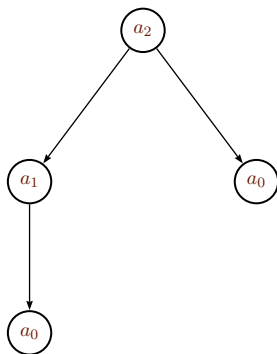
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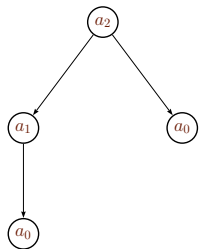
EXAMPLE 2:  $a_3 S S a_2 S a_0$

EXAMPLE:



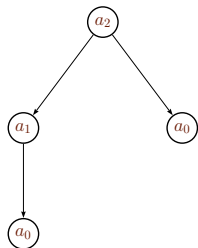
$$P = a_2 a_1 a_0 a_0$$

## SNPDA



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$$\begin{aligned} a_0 | S &\mapsto \varepsilon \\ a_1 | S &\mapsto S \\ a_2 | S &\mapsto SS \end{aligned}$$


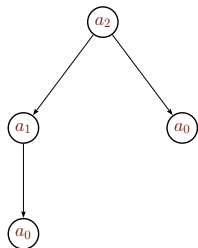
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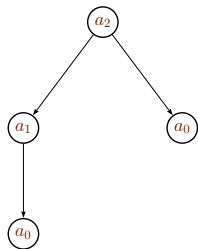
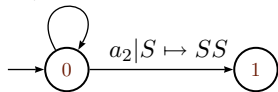


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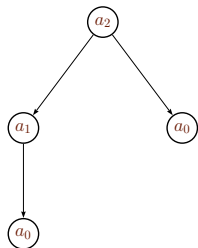
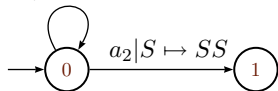
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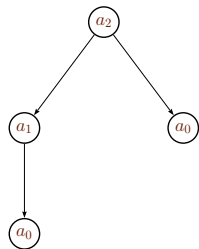
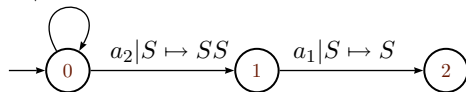
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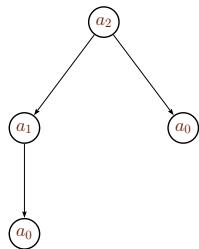
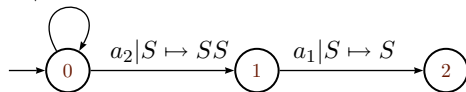
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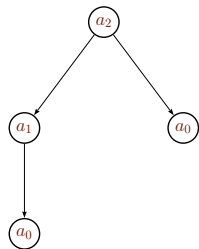
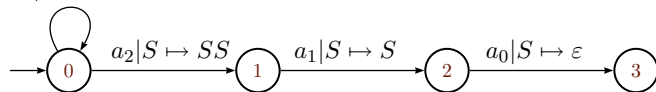
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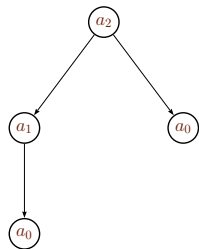
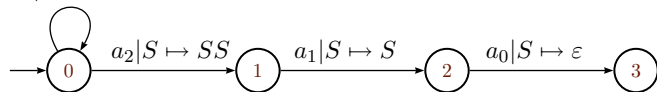
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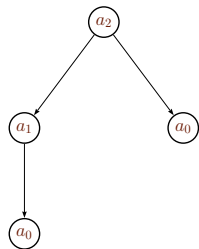
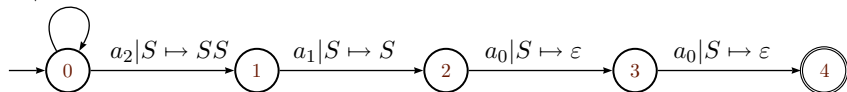
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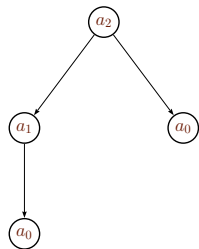
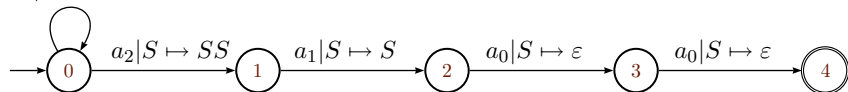
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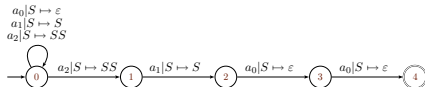
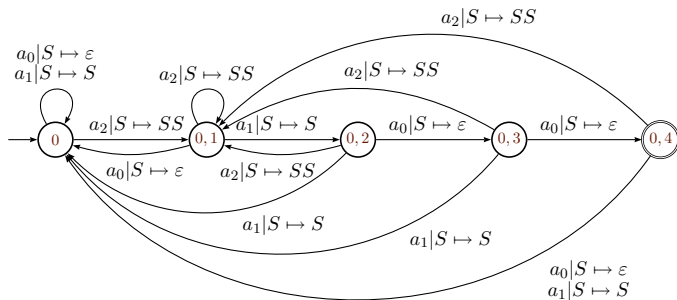
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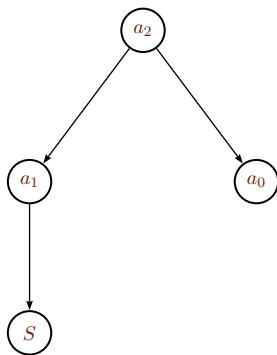
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DETERMINISTIC SEARCHING PUSHDOWN  
AUTOMATON

$$P = a_2 a_1 a_0 a_0$$

# TREE PATTERNS (TEMPLATES)

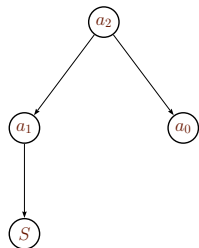
EXAMPLE 1:



$$P = a_2 a_1 s a_0$$

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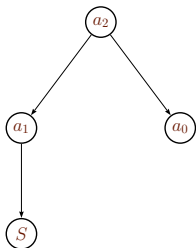
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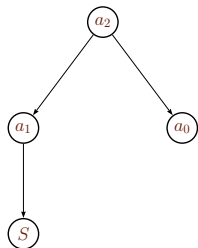
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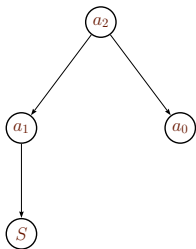
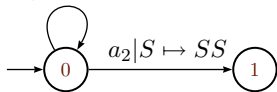
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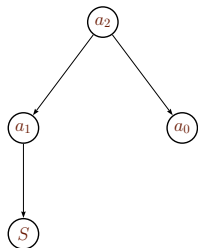
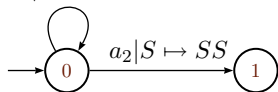
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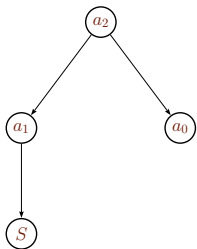
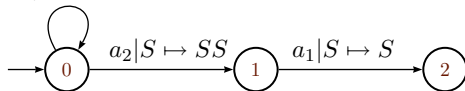
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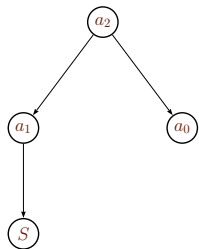
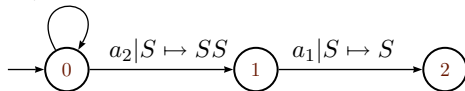
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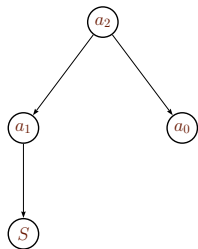
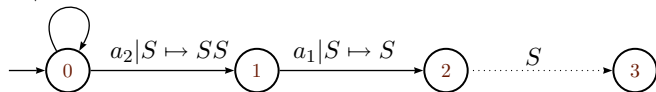
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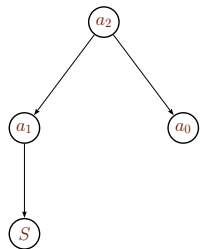
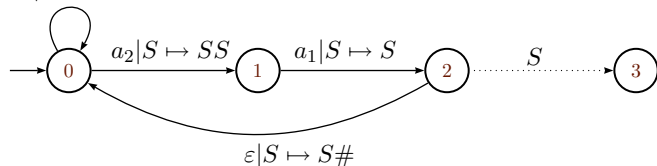
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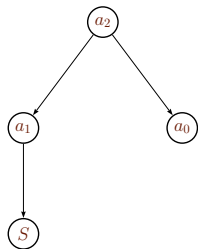
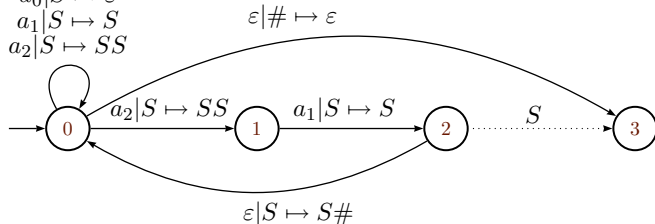


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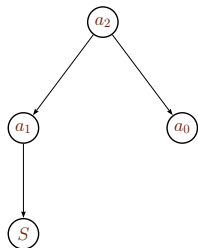
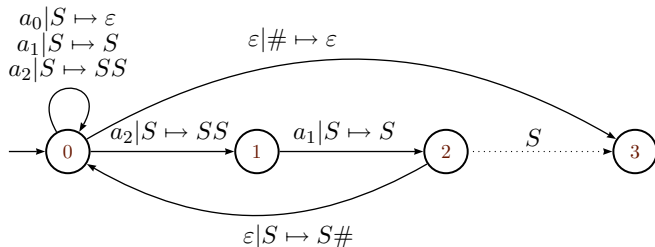


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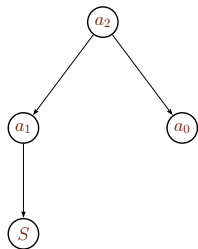
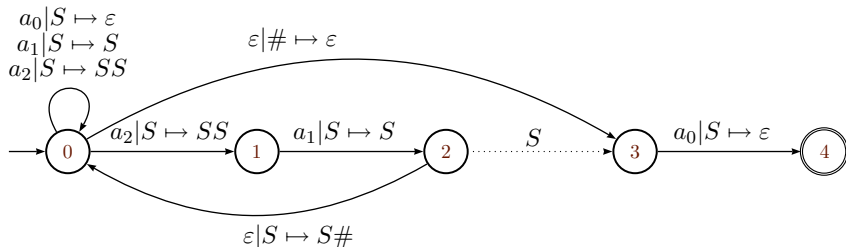
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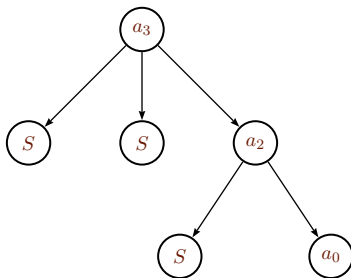
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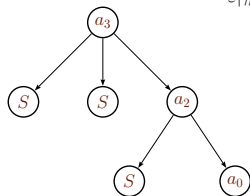
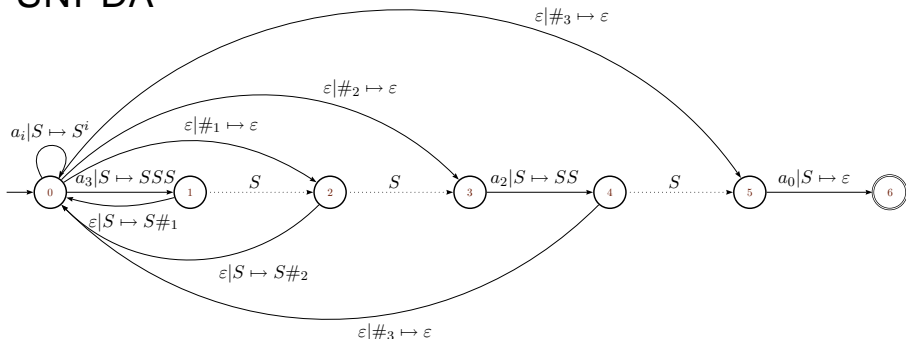
## EXAMPLE 2:



$$P = a_3 S S a_2 S a_0$$

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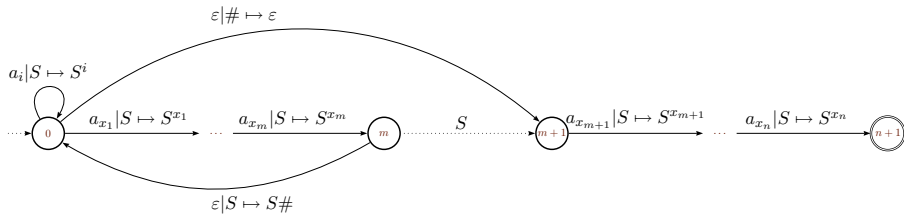
## SNPDA



$$P = a_3SSa_2Sa_0$$

# CAVEAT

$$\text{PATTERN } P = a_{x_1} \dots a_{x_m} S a_{x_{m+1}} \dots a_{x_n}$$

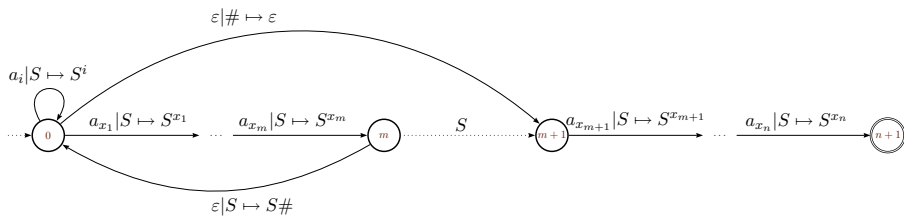


TWO WAYS TO PROCESS  $a_{x_1} \dots a_{x_m}$

PUSHDOWN STORE  $\left\{ \begin{array}{l} \# \\ \epsilon \end{array} \right.$

# CAVEAT

PATTERN  $P = a_{x_1} \dots a_{x_m} S a_{x_{m+1}} \dots a_{x_n}$

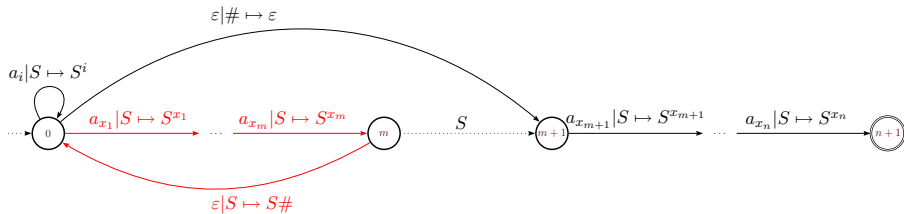


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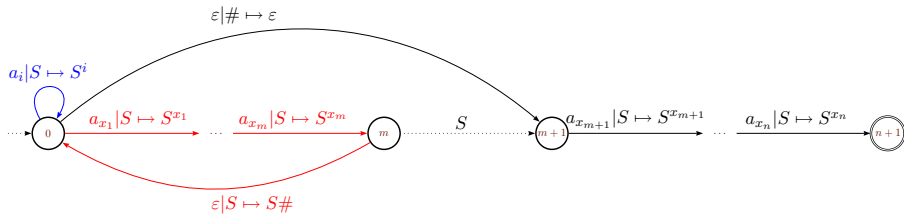


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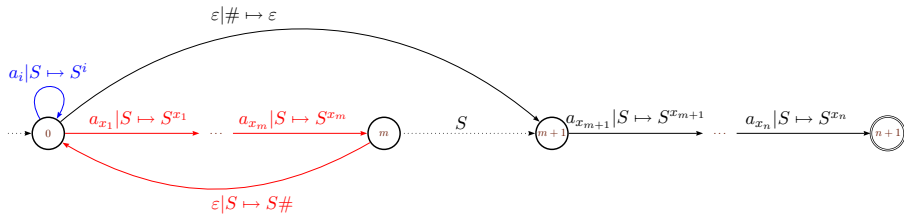


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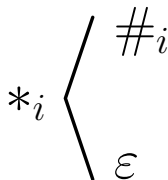
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TWO WAYS TO PROCESS  $a_{x_1} \dots a_{x_m}$

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USAGE OF NEW PUSHDOWN STORE (PDS) SYMBOLS...

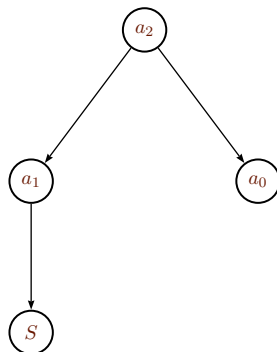


...SIMULATING THE TWO CASES.

# EXAMPLE

TEMPLATE IN PREFIX  
NOTATION:

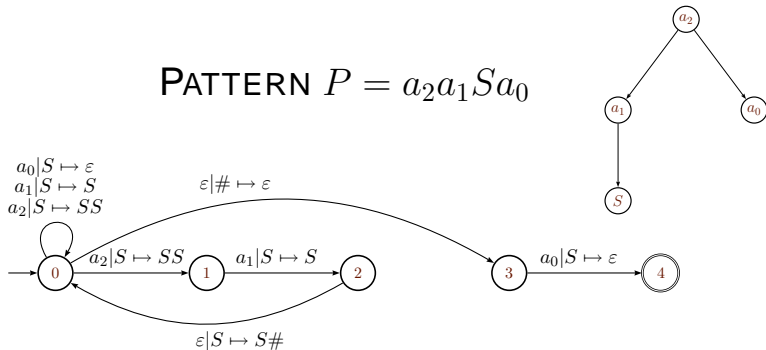
$$P = a_2 a_1 S a_0$$



# EXAMPLE

## ELIMINATION OF $\varepsilon$ -TRANSITION FROM STATE 2 TO STATE 0

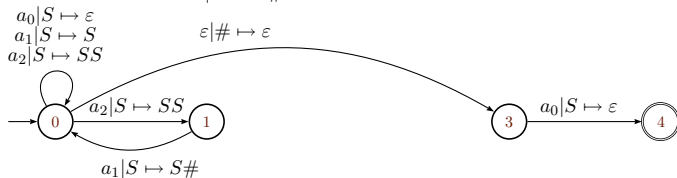
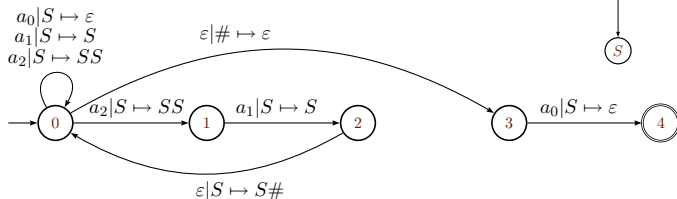
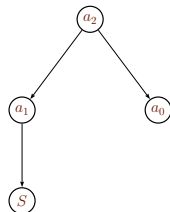
PATTERN  $P = a_2a_1Sa_0$



# EXAMPLE

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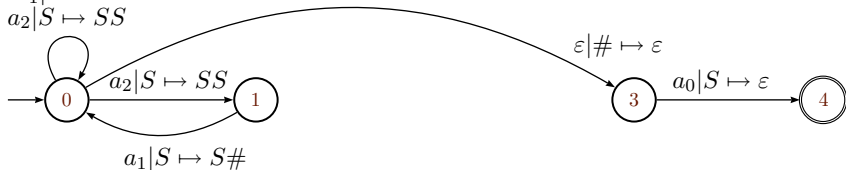
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# EXAMPLE

PATTERN  $P = a_2a_1Sa_0$

$a_0|S \mapsto \varepsilon$   
 $a_1|S \mapsto S$   
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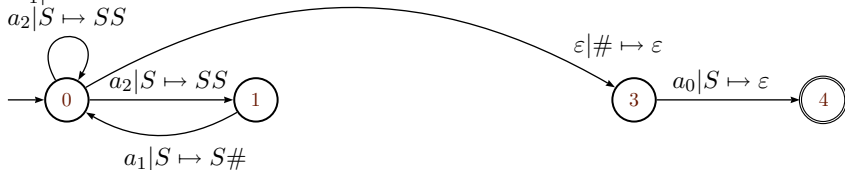
	$a_0$	$a_1$	$a_2$	$\varepsilon$
0	$0 S \mapsto \varepsilon$	$0 S \mapsto S$	$0 S \mapsto SS$ $1 S \mapsto SS$	$3 \# \mapsto \varepsilon$
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4				

	$a_0$	$a_1$	$a_2$	$\varepsilon$
[0]	$[0] S \mapsto \varepsilon$	$[0] S \mapsto S$	$[0, 1] S \mapsto SS$	$[0, 3] * \mapsto \varepsilon$
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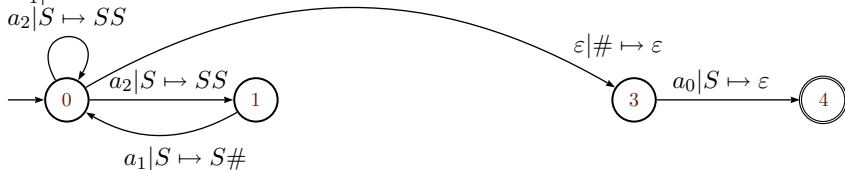
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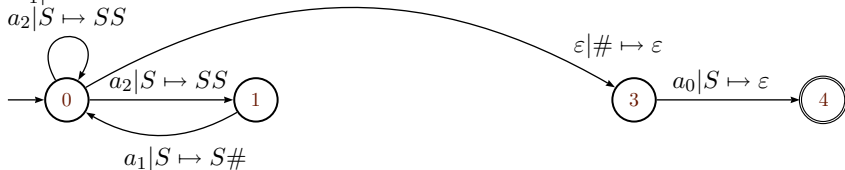
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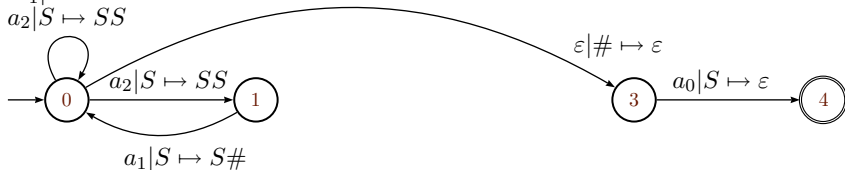
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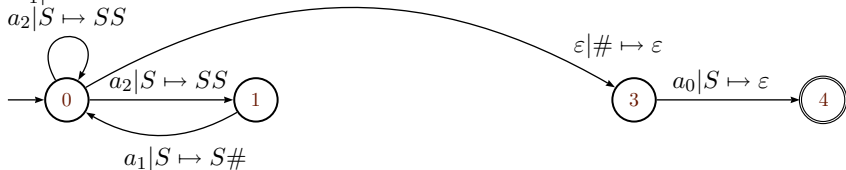
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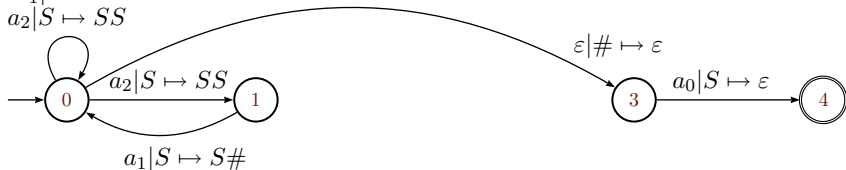
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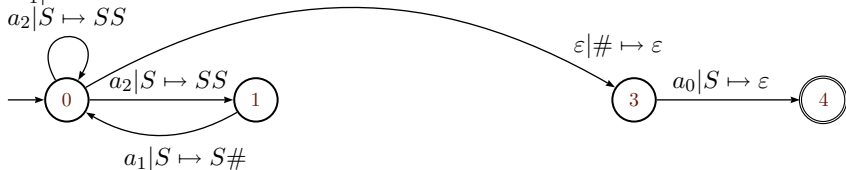
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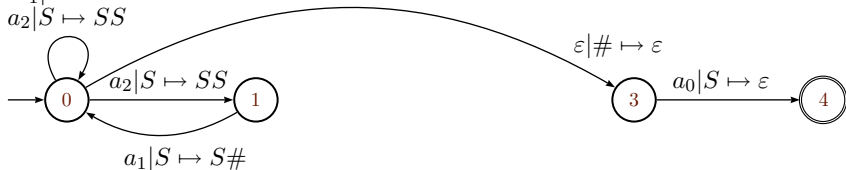
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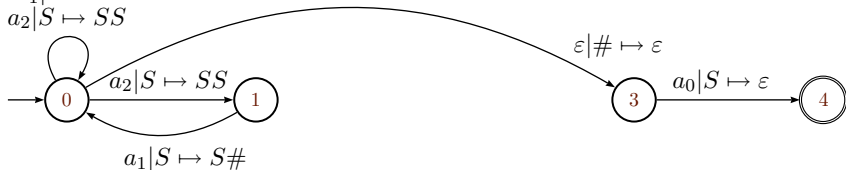
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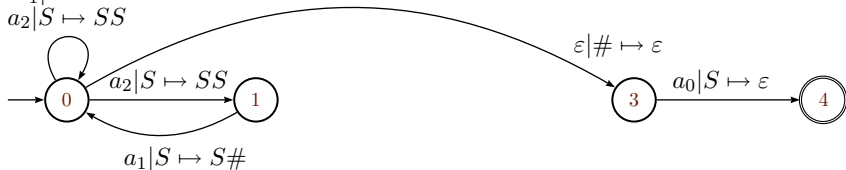
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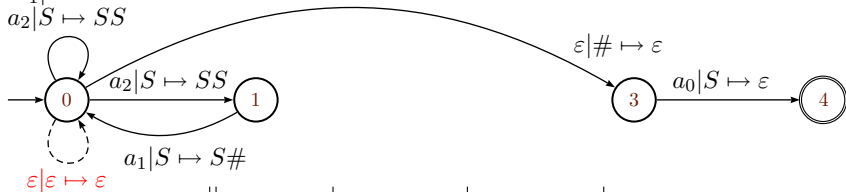
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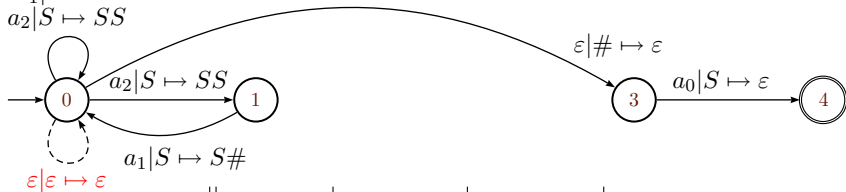
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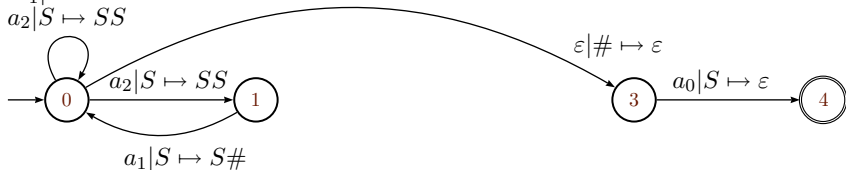
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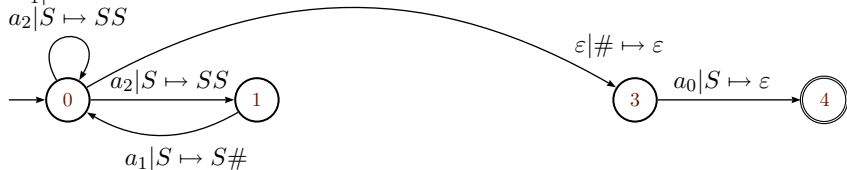
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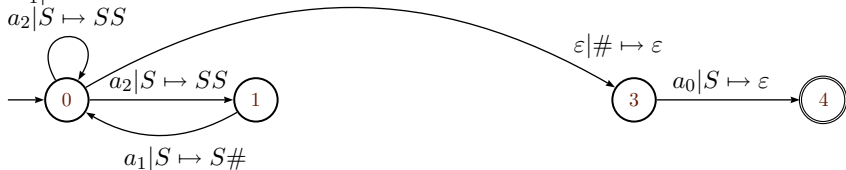
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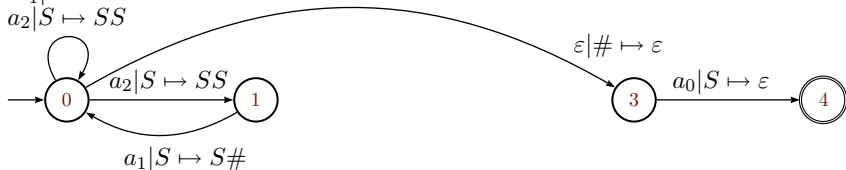
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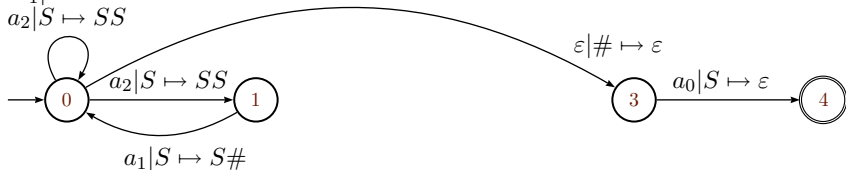
	$a_0$	$a_1$	$a_2$	$\varepsilon$
0	$0 S \mapsto \varepsilon$	$0 S \mapsto S$	$0 S \mapsto SS$ $1 S \mapsto SS$	$3 \# \mapsto \varepsilon$
1		$0 S \mapsto S\#$		
3	$4 S \mapsto \varepsilon$			
4				

	$a_0$	$a_1$	$a_2$	$\varepsilon$
[0]	$[0] S \mapsto \varepsilon$	$[0] S \mapsto S$	$[0, 1] S \mapsto SS$	$[0, 3] * \mapsto \varepsilon$
[0, 1]	$[0] S \mapsto \varepsilon$	$[0] S \mapsto S*$	$[0, 1] S \mapsto SS$	$[0, 3] * \mapsto \varepsilon$
[0, 3]	$[0, 4] S \mapsto \varepsilon$	$[0] S \mapsto S$	$[0, 1] S \mapsto SS$	$[0, 3] * \mapsto \varepsilon$
[0, 4]	$[0] S \mapsto \varepsilon$	$[0] S \mapsto S$	$[0, 1] S \mapsto SS$	$[0, 3] * \mapsto \varepsilon$

# EXAMPLE

PATTERN  $P = a_2a_1Sa_0$

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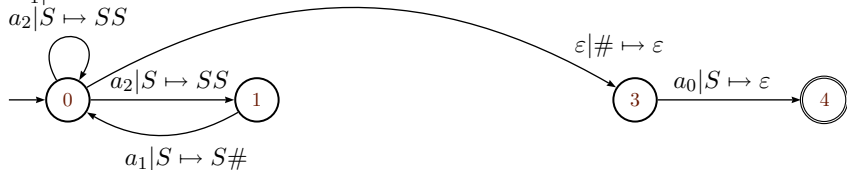
	$a_0$	$a_1$	$a_2$	$\varepsilon$
0	$0 S \mapsto \varepsilon$	$0 S \mapsto S$	$0 S \mapsto SS$ $1 S \mapsto SS$	$3 \# \mapsto \varepsilon$
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	$a_0$	$a_1$	$a_2$	$\varepsilon$
[0]	$[0] S \mapsto \varepsilon$	$[0] S \mapsto S$	$[0, 1] S \mapsto SS$	$[0, 3] * \mapsto \varepsilon$
[0, 1]	$[0] S \mapsto \varepsilon$	$[0] S \mapsto S$	$[0, 1] S \mapsto SS$	$[0, 3] * \mapsto \varepsilon$
[0, 3]	$[0, 4] S \mapsto \varepsilon$	$[0] S \mapsto S$	$[0, 1] S \mapsto SS$	$[0, 3] * \mapsto \varepsilon$
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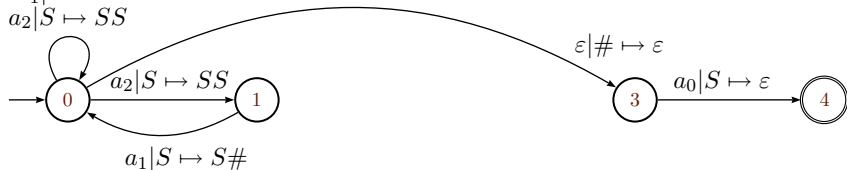
	$a_0$	$a_1$	$a_2$	$\varepsilon$
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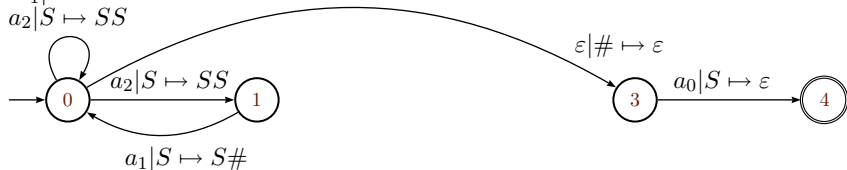
	$a_0$	$a_1$	$a_2$	$\varepsilon$
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	$a_0$	$a_1$	$a_2$	$\varepsilon$
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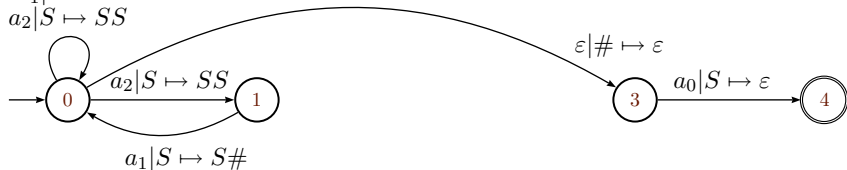
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[0]	$[0] S \mapsto \varepsilon$	$[0] S \mapsto S$	$[0, 1] S \mapsto SS$	$[0, 3] * \mapsto \varepsilon$
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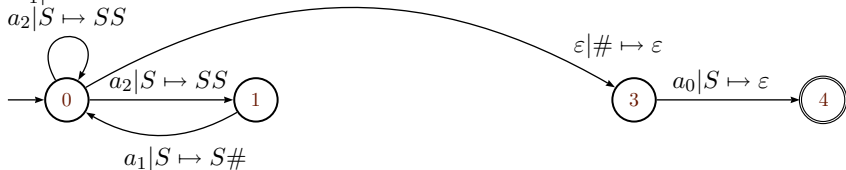
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0	$0 S \mapsto \varepsilon$	$0 S \mapsto S$	$0 S \mapsto SS$ $1 S \mapsto SS$	$3 \# \mapsto \varepsilon$
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[0]	$[0] S \mapsto \varepsilon$	$[0] S \mapsto S$	$[0, 1] S \mapsto SS$	$[0, 3] * \mapsto \varepsilon$
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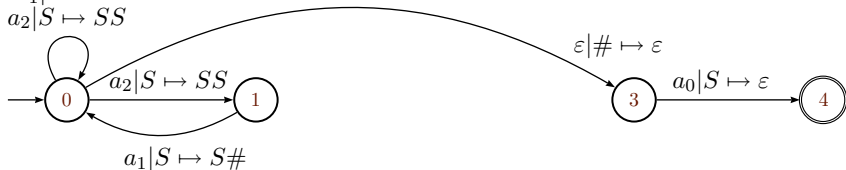
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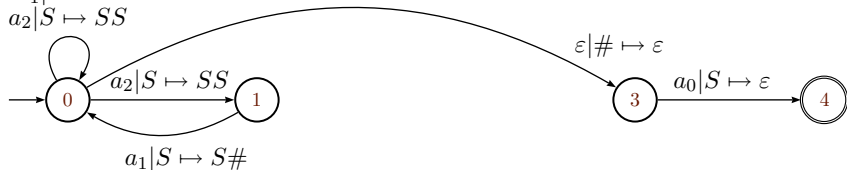
	$a_0$	$a_1$	$a_2$	$\varepsilon$
0	$0 S \mapsto \varepsilon$	$0 S \mapsto S$	$0 S \mapsto SS$ $1 S \mapsto SS$	$3 \# \mapsto \varepsilon$
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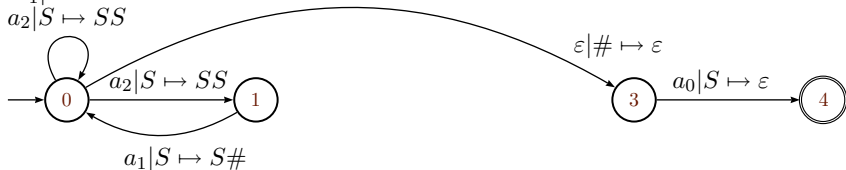
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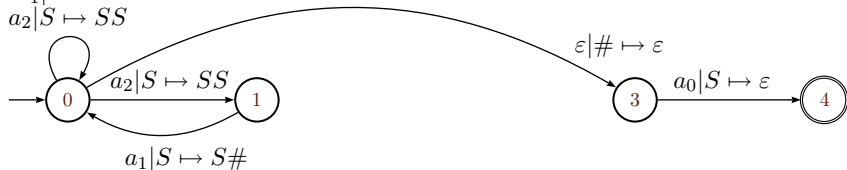
	$a_0$	$a_1$	$a_2$	$\varepsilon$
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[0, 1]	$[0] S \mapsto \varepsilon$	$[0] S \mapsto S*$	$[0, 1] S \mapsto SS$	$[0, 3] * \mapsto \varepsilon$
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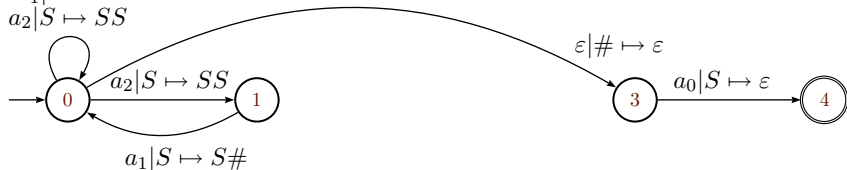
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	$a_0$	$a_1$	$a_2$	$\varepsilon$
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[0, 1]	$[0] S \mapsto \varepsilon$	$[0] S \mapsto S*$	$[0, 1] S \mapsto SS$	$[0, 3] * \mapsto \varepsilon$
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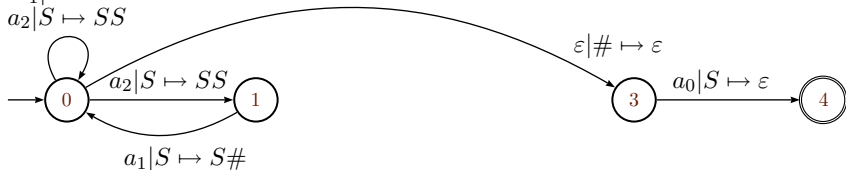
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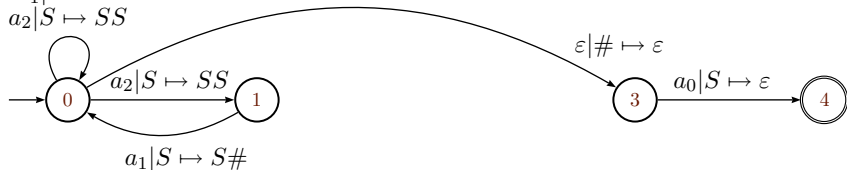
	$a_0$	$a_1$	$a_2$	$\varepsilon$
0	$0 S \mapsto \varepsilon$	$0 S \mapsto S$	$0 S \mapsto SS$ $1 S \mapsto SS$	$3 \# \mapsto \varepsilon$
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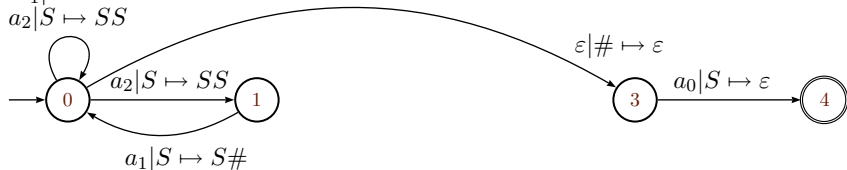
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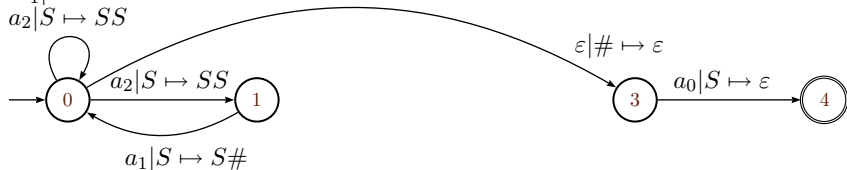
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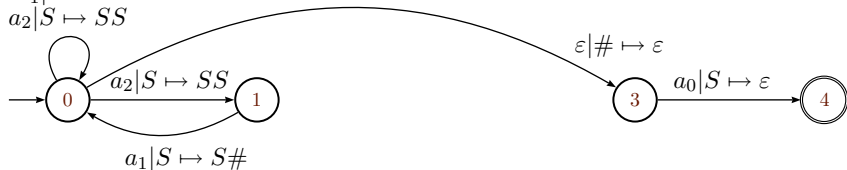
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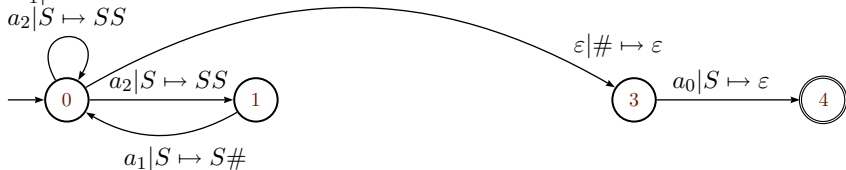
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[0, 4]	$[0] S \mapsto \varepsilon$	$[0] S \mapsto S$	$[0, 1] S \mapsto SS$	$[0, 3] * \mapsto \varepsilon$

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PATTERN  $P = a_2a_1Sa_0$

$a_0|S \mapsto \varepsilon$   
 $a_1|S \mapsto S$   
 $a_2|S \mapsto SS$



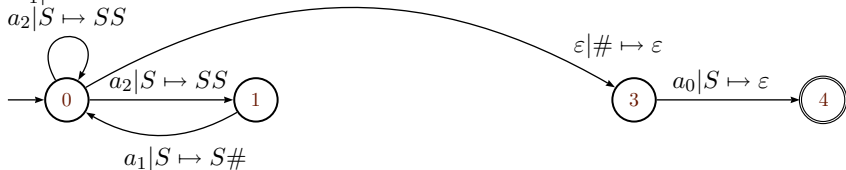
	$a_0$	$a_1$	$a_2$	$\varepsilon$
0	$0 S \mapsto \varepsilon$	$0 S \mapsto S$	$0 S \mapsto SS$ $1 S \mapsto SS$	$3 \# \mapsto \varepsilon$
1		$0 S \mapsto S\#$		
3	$4 S \mapsto \varepsilon$			
4				

	$a_0$	$a_1$	$a_2$	$\varepsilon$
[0]	$[0] S \mapsto \varepsilon$	$[0] S \mapsto S$	$[0, 1] S \mapsto SS$	$[0, 3] * \mapsto \varepsilon$
[0, 1]	$[0] S \mapsto \varepsilon$	$[0] S \mapsto S*$	$[0, 1] S \mapsto SS$	$[0, 3] * \mapsto \varepsilon$
[0, 3]	$[0, 4] S \mapsto \varepsilon$	$[0] S \mapsto S$	$[0, 1] S \mapsto SS$	$[0, 3] * \mapsto \varepsilon$
[0, 4]	$[0] S \mapsto \varepsilon$	$[0] S \mapsto S$	$[0, 1] S \mapsto SS$	$[0, 3] * \mapsto \varepsilon$

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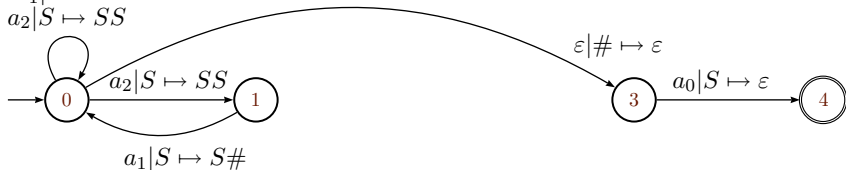
	$a_0$	$a_1$	$a_2$	$\varepsilon$
0	$0 S \mapsto \varepsilon$	$0 S \mapsto S$	$0 S \mapsto SS$ $1 S \mapsto SS$	$3 \# \mapsto \varepsilon$
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	$a_0$	$a_1$	$a_2$	$\varepsilon$
[0]	$[0] S \mapsto \varepsilon$	$[0] S \mapsto S$	$[0, 1] S \mapsto SS$	$[0, 3] * \mapsto \varepsilon$
[0, 1]	$[0] S \mapsto \varepsilon$	$[0] S \mapsto S*$	$[0, 1] S \mapsto SS$	$[0, 3] * \mapsto \varepsilon$
[0, 3]	$[0, 4] S \mapsto \varepsilon$	$[0] S \mapsto S$	$[0, 1] S \mapsto SS$	$[0, 3] * \mapsto \varepsilon$
[0, 4]	$[0] S \mapsto \varepsilon$	$[0] S \mapsto S$	$[0, 1] S \mapsto SS$	$[0, 3] * \mapsto \varepsilon$

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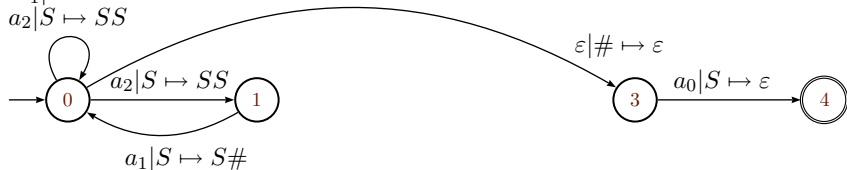
	$a_0$	$a_1$	$a_2$	$\varepsilon$
0	$0 S \mapsto \varepsilon$	$0 S \mapsto S$	$0 S \mapsto SS$ $1 S \mapsto SS$	$3 \# \mapsto \varepsilon$
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	$a_0$	$a_1$	$a_2$	$\varepsilon$
[0]	$[0] S \mapsto \varepsilon$	$[0] S \mapsto S$	$[0, 1] S \mapsto SS$	$[0, 3] * \mapsto \varepsilon$
[0, 1]	$[0] S \mapsto \varepsilon$	$[0] S \mapsto S*$	$[0, 1] S \mapsto SS$	$[0, 3] * \mapsto \varepsilon$
[0, 3]	$[0, 4] S \mapsto \varepsilon$	$[0] S \mapsto S$	$[0, 1] S \mapsto SS$	$[0, 3] * \mapsto \varepsilon$
[0, 4]	$[0] S \mapsto \varepsilon$	$[0] S \mapsto S$	$[0, 1] S \mapsto SS$	$[0, 3] * \mapsto \varepsilon$

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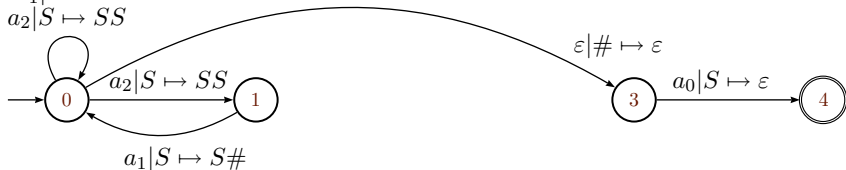
	$a_0$	$a_1$	$a_2$	$\varepsilon$
0	$0 S \mapsto \varepsilon$	$0 S \mapsto S$	$0 S \mapsto SS$ $1 S \mapsto SS$	$3 \# \mapsto \varepsilon$
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4				

	$a_0$	$a_1$	$a_2$	$\varepsilon$
[0]	$[0] S \mapsto \varepsilon$	$[0] S \mapsto S$	$[0, 1] S \mapsto SS$	$[0, 3] * \mapsto \varepsilon$
[0, 1]	$[0] S \mapsto \varepsilon$	$[0] S \mapsto S*$	$[0, 1] S \mapsto SS$	$[0, 3] * \mapsto \varepsilon$
[0, 3]	$[0, 4] S \mapsto \varepsilon$	$[0] S \mapsto S$	$[0, 1] S \mapsto SS$	$[0, 3] * \mapsto \varepsilon$
[0, 4]	$[0] S \mapsto \varepsilon$	$[0] S \mapsto S$	$[0, 1] S \mapsto SS$	$[0, 3] * \mapsto \varepsilon$

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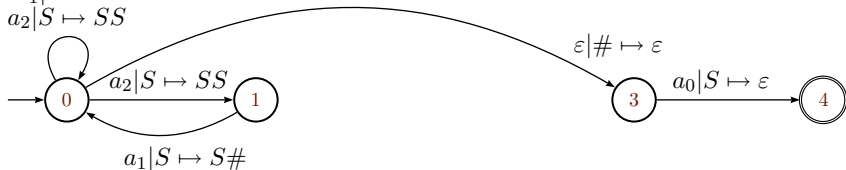
	$a_0$	$a_1$	$a_2$	$\varepsilon$
0	$0 S \mapsto \varepsilon$	$0 S \mapsto S$	$0 S \mapsto SS$ $1 S \mapsto SS$	$3 \# \mapsto \varepsilon$
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	$a_0$	$a_1$	$a_2$	$\varepsilon$
[0]	$[0] S \mapsto \varepsilon$	$[0] S \mapsto S$	$[0, 1] S \mapsto SS$	$[0, 3] * \mapsto \varepsilon$
[0, 1]	$[0] S \mapsto \varepsilon$	$[0] S \mapsto S*$	$[0, 1] S \mapsto SS$	$[0, 3] * \mapsto \varepsilon$
[0, 3]	$[0, 4] S \mapsto \varepsilon$	$[0] S \mapsto S$	$[0, 1] S \mapsto SS$	$[0, 3] * \mapsto \varepsilon$
[0, 4]	$[0] S \mapsto \varepsilon$	$[0] S \mapsto S$	$[0, 1] S \mapsto SS$	$[0, 3] * \mapsto \varepsilon$

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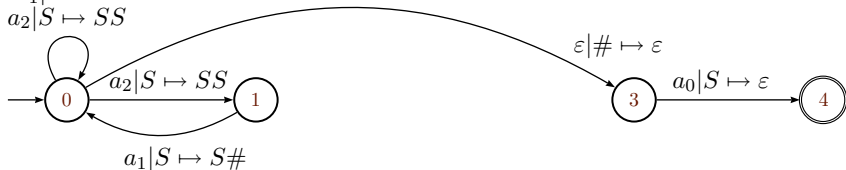
	$a_0$	$a_1$	$a_2$	$\varepsilon$
0	$0 S \mapsto \varepsilon$	$0 S \mapsto S$	$0 S \mapsto SS$ $1 S \mapsto SS$	$3 \# \mapsto \varepsilon$
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	$a_0$	$a_1$	$a_2$	$\varepsilon$
[0]	$[0] S \mapsto \varepsilon$	$[0] S \mapsto S$	$[0, 1] S \mapsto SS$	$[0, 3] * \mapsto \varepsilon$
[0, 1]	$[0] S \mapsto \varepsilon$	$[0] S \mapsto S*$	$[0, 1] S \mapsto SS$	$[0, 3] * \mapsto \varepsilon$
[0, 3]	$[0, 4] S \mapsto \varepsilon$	$[0] S \mapsto S$	$[0, 1] S \mapsto SS$	$[0, 3] * \mapsto \varepsilon$
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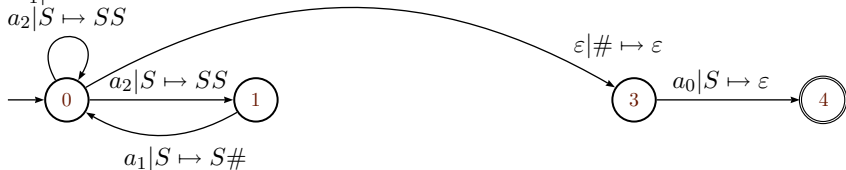
	$a_0$	$a_1$	$a_2$	$\varepsilon$
0	$0 S \mapsto \varepsilon$	$0 S \mapsto S$	$0 S \mapsto SS$ $1 S \mapsto SS$	$3 \# \mapsto \varepsilon$
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	$a_0$	$a_1$	$a_2$	$\varepsilon$
[0]	$[0] S \mapsto \varepsilon$	$[0] S \mapsto S$	$[0, 1] S \mapsto SS$	$[0, 3] * \mapsto \varepsilon$
[0, 1]	$[0] S \mapsto \varepsilon$	$[0] S \mapsto S*$	$[0, 1] S \mapsto SS$	$[0, 3] * \mapsto \varepsilon$
[0, 3]	$[0, 4] S \mapsto \varepsilon$	$[0] S \mapsto S$	$[0, 1] S \mapsto SS$	$[0, 3] * \mapsto \varepsilon$
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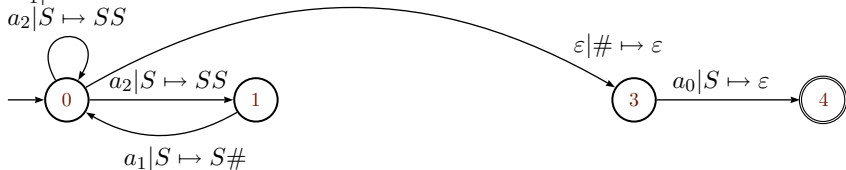
	$a_0$	$a_1$	$a_2$	$\varepsilon$
0	$0 S \mapsto \varepsilon$	$0 S \mapsto S$	$0 S \mapsto SS$ $1 S \mapsto SS$	$3 \# \mapsto \varepsilon$
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	$a_0$	$a_1$	$a_2$	$\varepsilon$
[0]	$[0] S \mapsto \varepsilon$	$[0] S \mapsto S$	$[0, 1] S \mapsto SS$	$[0, 3]* \mapsto \varepsilon$
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[0, 3]	$[0, 4] S \mapsto \varepsilon$	$[0] S \mapsto S$	$[0, 1] S \mapsto SS$	$[0, 3]* \mapsto \varepsilon$
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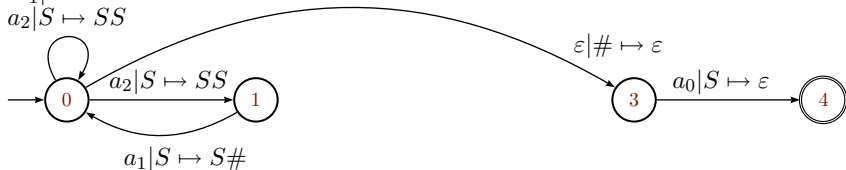
	$a_0$	$a_1$	$a_2$	$\varepsilon$
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	$a_0$	$a_1$	$a_2$	$\varepsilon$
[0]	$[0] S \mapsto \varepsilon$	$[0] S \mapsto S$	$[0, 1] S \mapsto SS$	$[0, 3] * \mapsto \varepsilon$
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[0, 3]	$[0, 4] S \mapsto \varepsilon$	$[0] S \mapsto S$	$[0, 1] S \mapsto SS$	$[0, 3] * \mapsto \varepsilon$
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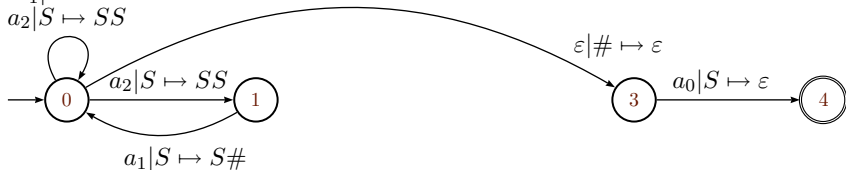
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	$a_0$	$a_1$	$a_2$	$\varepsilon$
[0]	$[0] S \mapsto \varepsilon$	$[0] S \mapsto S$	$[0, 1] S \mapsto SS$	$[0, 3] * \mapsto \varepsilon$
[0, 1]	$[0] S \mapsto \varepsilon$	$[0] S \mapsto S*$	$[0, 1] S \mapsto SS$	$[0, 3] * \mapsto \varepsilon$
[0, 3]	$[0, 4] S \mapsto \varepsilon$	$[0] S \mapsto S$	$[0, 1] S \mapsto SS$	$[0, 3] * \mapsto \varepsilon$
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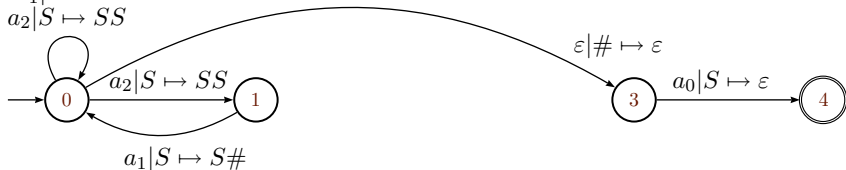
	$a_0$	$a_1$	$a_2$	$\varepsilon$
0	$0 S \mapsto \varepsilon$	$0 S \mapsto S$	$0 S \mapsto SS$ $1 S \mapsto SS$	$3 \# \mapsto \varepsilon$
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	$a_0$	$a_1$	$a_2$	$\varepsilon$
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[0, 3]	$[0, 4] S \mapsto \varepsilon$	$[0] S \mapsto S$	$[0, 1] S \mapsto SS$	$[0, 3] * \mapsto \varepsilon$
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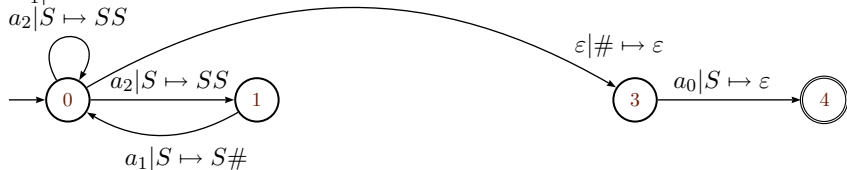
	$a_0$	$a_1$	$a_2$	$\varepsilon$
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	$a_0$	$a_1$	$a_2$	$\varepsilon$
[0]	$[0] S \mapsto \varepsilon$	$[0] S \mapsto S$	$[0, 1] S \mapsto SS$	$[0, 3] * \mapsto \varepsilon$
[0, 1]	$[0] S \mapsto \varepsilon$	$[0] S \mapsto S*$	$[0, 1] S \mapsto SS$	$[0, 3] * \mapsto \varepsilon$
[0, 3]	$[0, 4] S \mapsto \varepsilon$	$[0] S \mapsto S$	$[0, 1] S \mapsto SS$	$[0, 3] * \mapsto \varepsilon$
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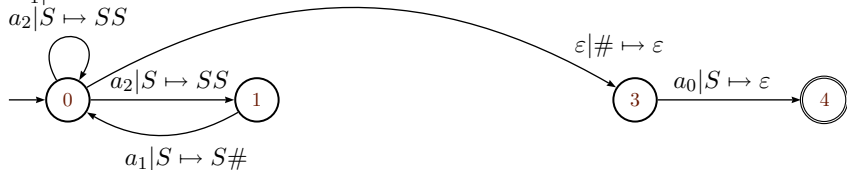
	$a_0$	$a_1$	$a_2$	$\varepsilon$
0	$0 S \mapsto \varepsilon$	$0 S \mapsto S$	$0 S \mapsto SS$ $1 S \mapsto SS$	$3 \# \mapsto \varepsilon$
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	$a_0$	$a_1$	$a_2$	$\varepsilon$
[0]	$[0] S \mapsto \varepsilon$	$[0] S \mapsto S$	$[0, 1] S \mapsto SS$	$[0, 3] * \mapsto \varepsilon$
[0, 1]	$[0] S \mapsto \varepsilon$	$[0] S \mapsto S*$	$[0, 1] S \mapsto SS$	$[0, 3] * \mapsto \varepsilon$
[0, 3]	$[0, 4] S \mapsto \varepsilon$	$[0] S \mapsto S$	$[0, 1] S \mapsto SS$	$[0, 3] * \mapsto \varepsilon$
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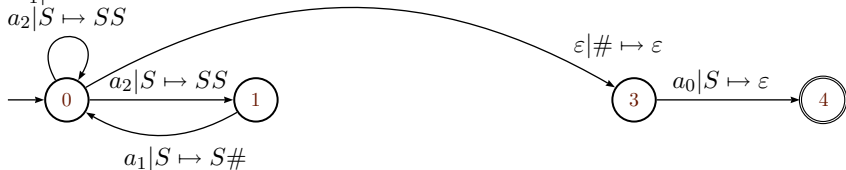
	$a_0$	$a_1$	$a_2$	$\varepsilon$
0	$0 S \mapsto \varepsilon$	$0 S \mapsto S$	$0 S \mapsto SS$ $1 S \mapsto SS$	$3 \# \mapsto \varepsilon$
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	$a_0$	$a_1$	$a_2$	$\varepsilon$
[0]	$[0] S \mapsto \varepsilon$	$[0] S \mapsto S$	$[0, 1] S \mapsto SS$	$[0, 3] * \mapsto \varepsilon$
[0, 1]	$[0] S \mapsto \varepsilon$	$[0] S \mapsto S*$	$[0, 1] S \mapsto SS$	$[0, 3] * \mapsto \varepsilon$
[0, 3]	$[0, 4] S \mapsto \varepsilon$	$[0] S \mapsto S$	$[0, 1] S \mapsto SS$	$[0, 3] * \mapsto \varepsilon$
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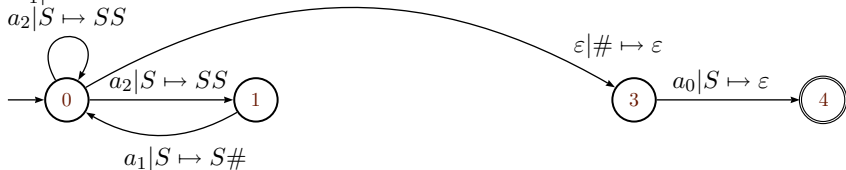
	$a_0$	$a_1$	$a_2$	$\varepsilon$
0	$0 S \mapsto \varepsilon$	$0 S \mapsto S$	$0 S \mapsto SS$ $1 S \mapsto SS$	$3 \# \mapsto \varepsilon$
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[0]	$[0] S \mapsto \varepsilon$	$[0] S \mapsto S$	$[0, 1] S \mapsto SS$	$[0, 3] * \mapsto \varepsilon$
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[0, 3]	$[0, 4] S \mapsto \varepsilon$	$[0] S \mapsto S$	$[0, 1] S \mapsto SS$	$[0, 3] * \mapsto \varepsilon$
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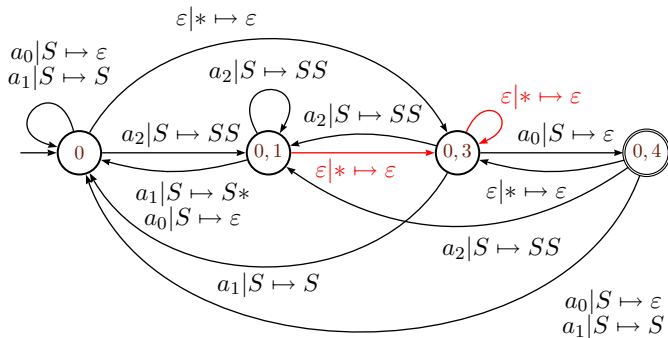
$a_0|S \mapsto \varepsilon$   
 $a_1|S \mapsto S$   
 $a_2|S \mapsto SS$



	$a_0$	$a_1$	$a_2$	$\varepsilon$
0	$0 S \mapsto \varepsilon$	$0 S \mapsto S$	$0 S \mapsto SS$ $1 S \mapsto SS$	$3 \# \mapsto \varepsilon$
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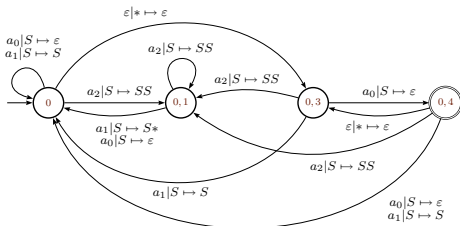
	$a_0$	$a_1$	$a_2$	$\varepsilon$
[0]	$[0] S \mapsto \varepsilon$	$[0] S \mapsto S$	$[0, 1] S \mapsto SS$	$[0, 3] * \mapsto \varepsilon$
[0, 1]	$[0] S \mapsto \varepsilon$	$[0] S \mapsto S*$	$[0, 1] S \mapsto SS$	$[0, 3] * \mapsto \varepsilon$
[0, 3]	$[0, 4] S \mapsto \varepsilon$	$[0] S \mapsto S$	$[0, 1] S \mapsto SS$	$[0, 3] * \mapsto \varepsilon$
[0, 4]	$[0] S \mapsto \varepsilon$	$[0] S \mapsto S$	$[0, 1] S \mapsto SS$	$[0, 3] * \mapsto \varepsilon$

# EXAMPLE



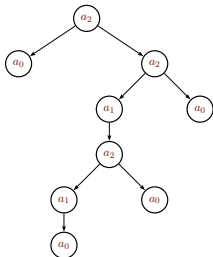
	$a_0$	$a_1$	$a_2$	$\epsilon$
[0]	$[0]   S \mapsto \epsilon$	$[0]   S \mapsto S$	$[0, 1]   S \mapsto SS$	$[0, 3]   * \mapsto \epsilon$
[0, 1]	$[0]   S \mapsto \epsilon$	$[0]   S \mapsto S^*$	$[0, 1]   S \mapsto SS$	$[0, 3]   * \mapsto \epsilon$
[0, 3]	$[0, 4]   S \mapsto \epsilon$	$[0]   S \mapsto S$	$[0, 1]   S \mapsto SS$	$[0, 3]   * \mapsto \epsilon$
[0, 4]	$[0]   S \mapsto \epsilon$	$[0]   S \mapsto S$	$[0, 1]   S \mapsto SS$	$[0, 3]   * \mapsto \epsilon$

# EXAMPLE



PDS    STATE    NODE

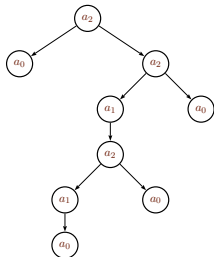
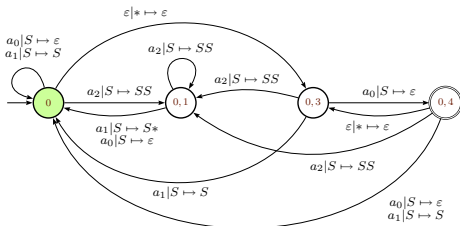
---



$$P = a_2 a_1 S a_0$$

$$a_2 a_0 a_2 a_1 a_2 a_1 a_0 a_0 a_0$$

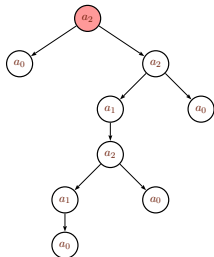
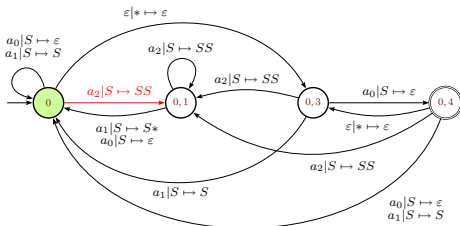
# EXAMPLE



$P = a_2a_1Sa_0$   
 $a_2a_0a_2a_1a_2a_1a_0a_0a_0$

PDS	STATE	NODE
S	[0]	

# EXAMPLE

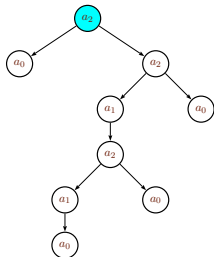
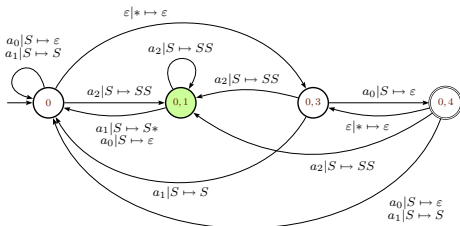


$$P = a_2 a_1 S a_0$$

$$a_2 a_0 a_2 a_1 a_2 a_1 a_0 a_0 a_0$$

PDS	STATE	NODE
S	[0]	a <sub>2</sub>

# EXAMPLE

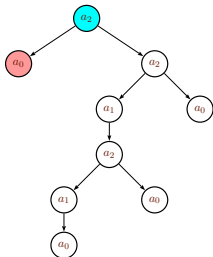
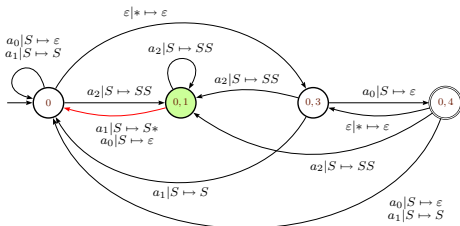


$$P = a_2 a_1 S a_0$$

$$a_2 a_0 a_2 a_1 a_2 a_1 a_0 a_0 a_0$$

PDS	STATE	NODE
$S$	$[0]$	$a_2$
$SS$	$[0, 1]$	

# EXAMPLE

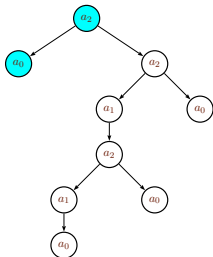
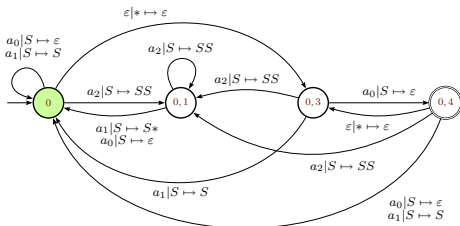


$$P = a_2 a_1 S a_0$$

$$a_2 a_0 a_2 a_1 a_2 a_1 a_0 a_0 a_0$$

PDS	STATE	NODE
$S$	$[0]$	$a_2$
$SS$	$[0, 1]$	$a_0$

# EXAMPLE

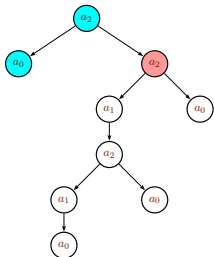
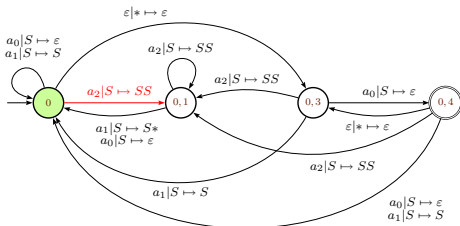


$P = a_2 a_1 S a_0$

$a_2 a_0 a_2 a_1 a_2 a_1 a_0 a_0 a_0$

PDS	STATE	NODE
$S$	$[0]$	$a_2$
$SS$	$[0, 1]$	$a_0$
$S$	$[0]$	

# EXAMPLE

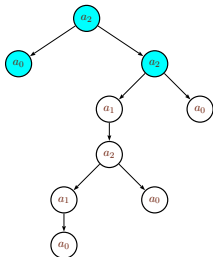
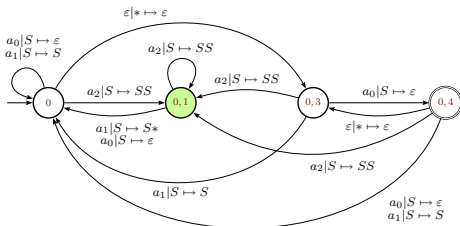


$P = a_2 a_1 S a_0$

$a_2 a_0 a_2 a_1 a_2 a_1 a_0 a_0 a_0$

PDS	STATE	NODE
$S$	$[0]$	$a_2$
$SS$	$[0, 1]$	$a_0$
$S$	$[0]$	$a_2$

# EXAMPLE

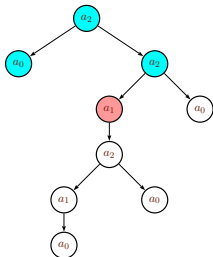
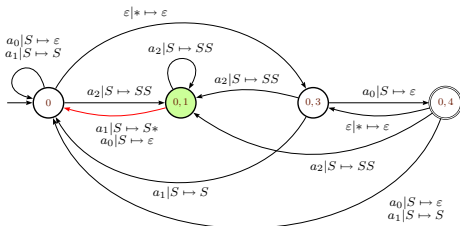


$P = a_2 a_1 S a_0$

$a_2 a_0 a_2 a_1 a_2 a_1 a_0 a_0 a_0$

PDS	STATE	NODE
$S$	$[0]$	$a_2$
$SS$	$[0, 1]$	$a_0$
$S$	$[0]$	$a_2$
$SS$	$[0, 1]$	

# EXAMPLE

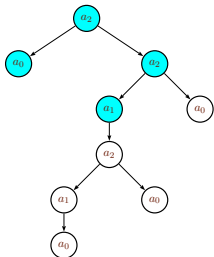
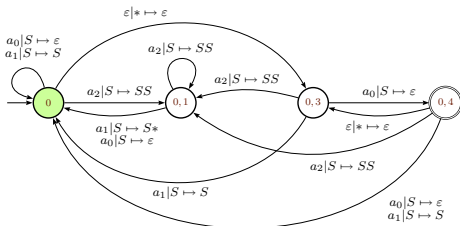


$P = a_2a_1Sa_0$

$a_2a_0a_2a_1a_2a_1a_0a_0a_0$

PDS	STATE	NODE
$S$	$[0]$	$a_2$
$SS$	$[0, 1]$	$a_0$
$S$	$[0]$	$a_2$
$SS$	$[0, 1]$	$a_1$

# EXAMPLE

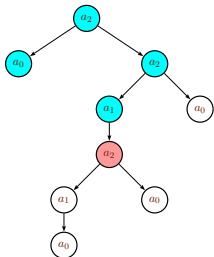
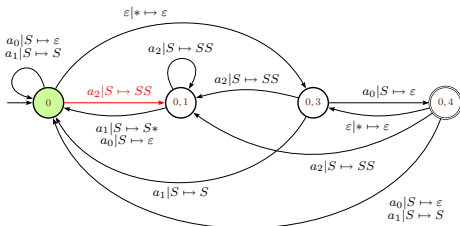


$$P = a_2a_1Sa_0$$

$$a_2a_0a_2a_1a_2a_1a_0a_0a_0$$

PDS	STATE	NODE
$S$	$[0]$	$a_2$
$SS$	$[0, 1]$	$a_0$
$S$	$[0]$	$a_2$
$SS$	$[0, 1]$	$a_1$
$S * S$	$[0]$	

# EXAMPLE

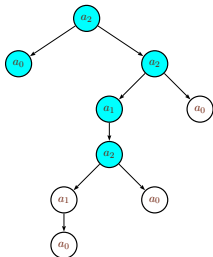
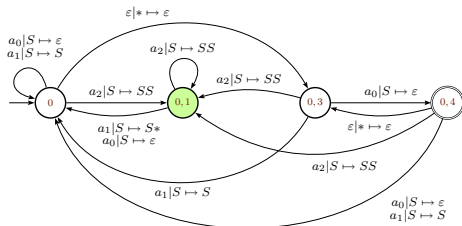


$P = a_2 a_1 S a_0$

$a_2 a_0 a_2 a_1 a_2 a_1 a_0 a_0 a_0$

PDS	STATE	NODE
$S$	$[0]$	$a_2$
$SS$	$[0, 1]$	$a_0$
$S$	$[0]$	$a_2$
$SS$	$[0, 1]$	$a_1$
$S * S$	$[0]$	$a_2$

# EXAMPLE

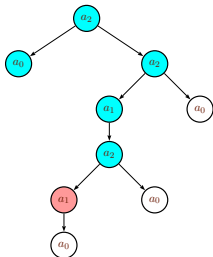
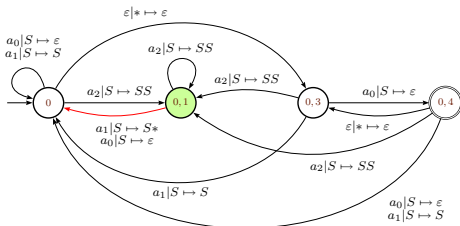


$P = a_2a_1Sa_0$

$a_2a_0a_2a_1a_2a_1a_0a_0a_0$

PDS	STATE	NODE
$S$	$[0]$	$a_2$
$SS$	$[0, 1]$	$a_0$
$S$	$[0]$	$a_2$
$SS$	$[0, 1]$	$a_1$
$S * S$	$[0]$	$a_2$
$SS * S$	$[0, 1]$	

# EXAMPLE

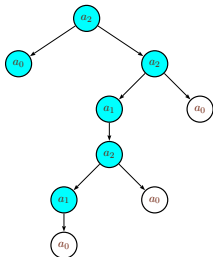
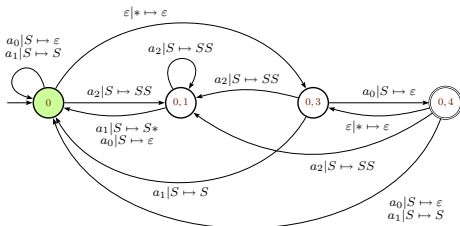


$P = a_2a_1Sa_0$

$a_2a_0a_2a_1a_2a_1a_0a_0a_0$

PDS	STATE	NODE
$S$	$[0]$	$a_2$
$SS$	$[0, 1]$	$a_0$
$S$	$[0]$	$a_2$
$SS$	$[0, 1]$	$a_1$
$S * S$	$[0]$	$a_2$
$SS * S$	$[0, 1]$	$a_1$

# EXAMPLE

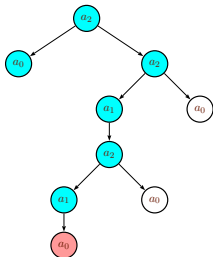
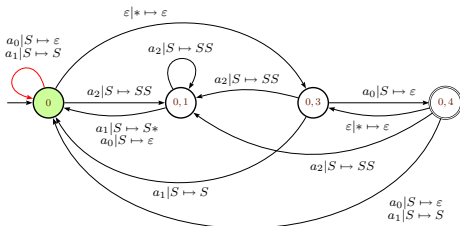


$$P = a_2 a_1 S a_0$$

$$a_2 a_0 a_2 a_1 a_2 a_1 a_0 a_0 a_0$$

PDS	STATE	NODE
$S$	$[0]$	$a_2$
$SS$	$[0, 1]$	$a_0$
$S$	$[0]$	$a_2$
$SS$	$[0, 1]$	$a_1$
$S * S$	$[0]$	$a_2$
$SS * S$	$[0, 1]$	$a_1$
$S * S * S$	$[0]$	

# EXAMPLE

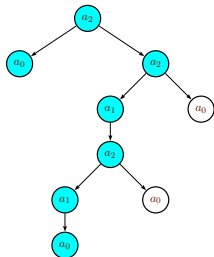
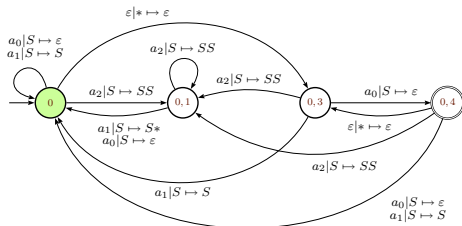


$$P = a_2a_1Sa_0$$

$$a_2a_0a_2a_1a_2a_1a_0a_0a_0$$

PDS	STATE	NODE
$S$	$[0]$	$a_2$
$SS$	$[0, 1]$	$a_0$
$S$	$[0]$	$a_2$
$SS$	$[0, 1]$	$a_1$
$S * S$	$[0]$	$a_2$
$SS * S$	$[0, 1]$	$a_1$
$S * S * S$	$[0]$	$a_0$

# EXAMPLE

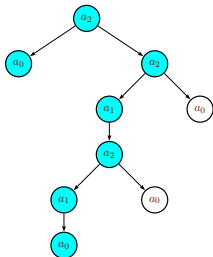
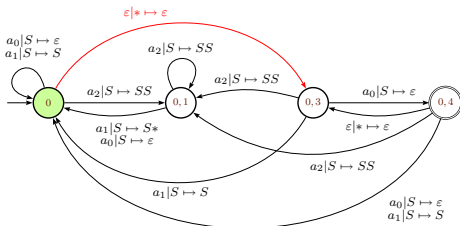


$P = a_2 a_1 S a_0$

$a_2 a_0 a_2 a_1 a_2 a_1 a_0 a_0 a_0$

PDS	STATE	NODE
$S$	$[0]$	$a_2$
$SS$	$[0, 1]$	$a_0$
$S$	$[0]$	$a_2$
$SS$	$[0, 1]$	$a_1$
$S * S$	$[0]$	$a_2$
$SS * S$	$[0, 1]$	$a_1$
$S * S * S$	$[0]$	$a_0$
$*S * S$	$[0]$	

# EXAMPLE

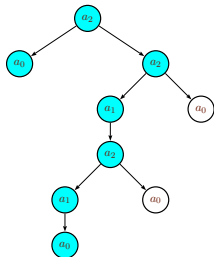
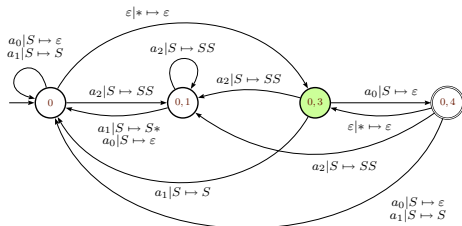


$P = a_2a_1Sa_0$

$a_2a_0a_2a_1a_2a_1a_0a_0a_0$

PDS	STATE	NODE
$S$	$[0]$	$a_2$
$SS$	$[0, 1]$	$a_0$
$S$	$[0]$	$a_2$
$SS$	$[0, 1]$	$a_1$
$S * S$	$[0]$	$a_2$
$SS * S$	$[0, 1]$	$a_1$
$S * S * S$	$[0]$	$a_0$
$*S * S$	$[0]$	

# EXAMPLE

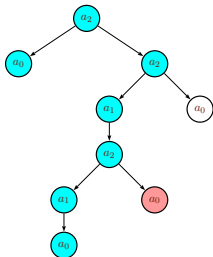
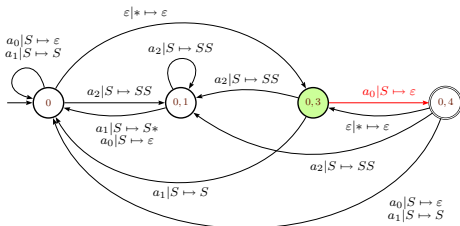


$P = a_2 a_1 S a_0$

$a_2 a_0 a_2 a_1 a_2 a_1 a_0 a_0 a_0$

PDS	STATE	NODE
$S$	$[0]$	$a_2$
$SS$	$[0, 1]$	$a_0$
$S$	$[0]$	$a_2$
$SS$	$[0, 1]$	$a_1$
$S * S$	$[0]$	$a_2$
$SS * S$	$[0, 1]$	$a_1$
$S * S * S$	$[0]$	$a_0$
$* S * S$	$[0]$	
$S * S$	$[0, 3]$	

# EXAMPLE

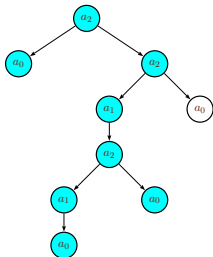
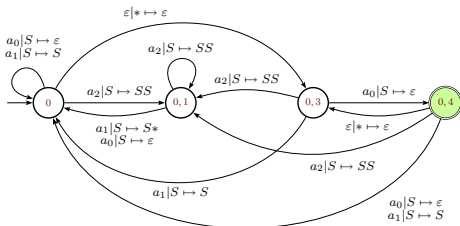


$P = a_2a_1Sa_0$

$a_2a_0a_2a_1a_2a_1a_0a_0a_0$

PDS	STATE	NODE
$S$	$[0]$	$a_2$
$SS$	$[0, 1]$	$a_0$
$S$	$[0]$	$a_2$
$SS$	$[0, 1]$	$a_1$
$S * S$	$[0]$	$a_2$
$SS * S$	$[0, 1]$	$a_1$
$S * S * S$	$[0]$	$a_0$
$*S * S$	$[0]$	
$S * S$	$[0, 3]$	$a_0$

# EXAMPLE

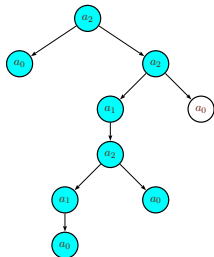
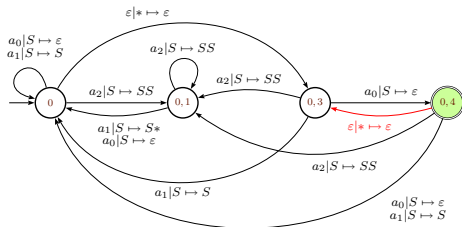


$P = a_2 a_1 S a_0$

$a_2 a_0 a_2 a_1 a_2 a_1 a_0 a_0 a_0$

PDS	STATE	NODE
$S$	$[0]$	$a_2$
$SS$	$[0, 1]$	$a_0$
$S$	$[0]$	$a_2$
$SS$	$[0, 1]$	$a_1$
$S * S$	$[0]$	$a_2$
$SS * S$	$[0, 1]$	$a_1$
$S * S * S$	$[0]$	$a_0$
$*S * S$	$[0]$	
$S * S$	$[0, 3]$	$a_0$
$*S$	$[0, 4]$	<b>MATCH</b>

# EXAMPLE

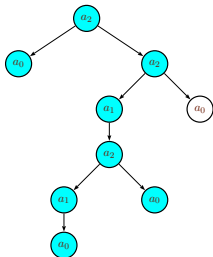
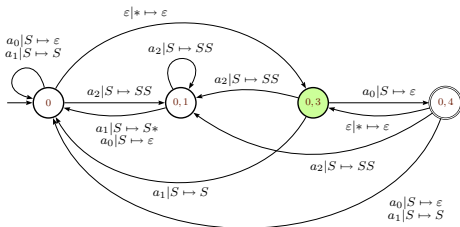


$P = a_2 a_1 S a_0$

$a_2 a_0 a_2 a_1 a_2 a_1 a_0 a_0 a_0$

PDS	STATE	NODE	
$S$	$[0]$	$a_2$	
$SS$	$[0, 1]$	$a_0$	
$S$	$[0]$	$a_2$	
$SS$	$[0, 1]$	$a_1$	
$S * S$	$[0]$	$a_2$	
$SS * S$	$[0, 1]$	$a_1$	
$S * S * S$	$[0]$	$a_0$	
$* S * S$	$[0]$		
$S * S$	$[0, 3]$	$a_0$	
$* S$	$[0, 4]$		MATCH

# EXAMPLE



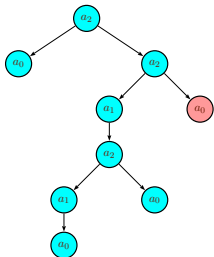
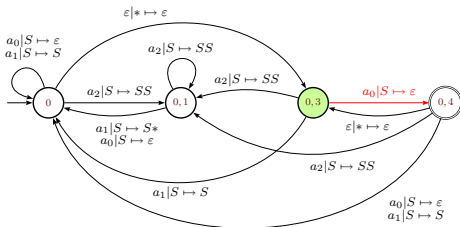
$P = a_2a_1Sa_0$

$a_2a_0a_2a_1a_2a_1a_0a_0a_0$

PDS	STATE	NODE
$S$	$[0]$	$a_2$
$SS$	$[0, 1]$	$a_0$
$S$	$[0]$	$a_2$
$SS$	$[0, 1]$	$a_1$
$S * S$	$[0]$	$a_2$
$SS * S$	$[0, 1]$	$a_1$
$S * S * S$	$[0]$	$a_0$
$*S * S$	$[0]$	
$S * S$	$[0, 3]$	$a_0$
$*S$	$[0, 4]$	
$S$	$[0, 3]$	

MATCH

# EXAMPLE



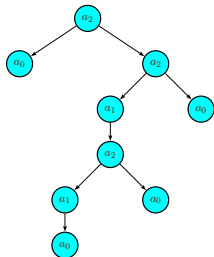
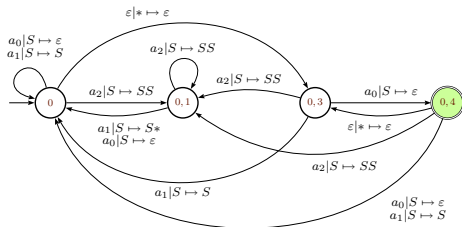
$P = a_2a_1Sa_0$

$a_2a_0a_2a_1a_2a_1a_0a_0a_0$

PDS	STATE	NODE
$S$	$[0]$	$a_2$
$SS$	$[0, 1]$	$a_0$
$S$	$[0]$	$a_2$
$SS$	$[0, 1]$	$a_1$
$S * S$	$[0]$	$a_2$
$SS * S$	$[0, 1]$	$a_1$
$S * S * S$	$[0]$	$a_0$
$*S * S$	$[0]$	
$S * S$	$[0, 3]$	$a_0$
$*S$	$[0, 4]$	
$S$	$[0, 3]$	$a_0$

MATCH

# EXAMPLE



$P = a_2 a_1 S a_0$

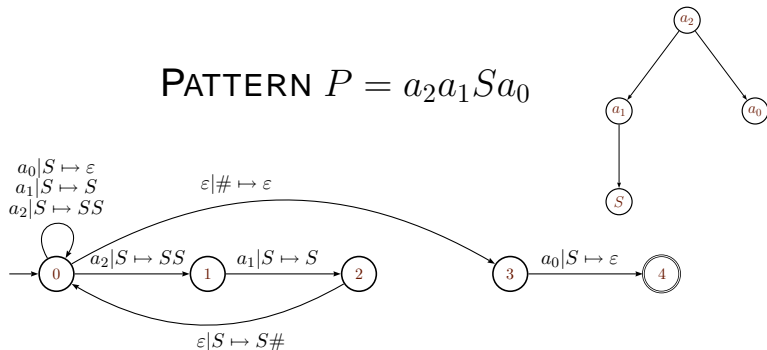
$a_2 a_0 a_2 a_1 a_2 a_1 a_0 a_0 a_0$

PDS	STATE	NODE
$S$	$[0]$	$a_2$
$SS$	$[0, 1]$	$a_0$
$S$	$[0]$	$a_2$
$SS$	$[0, 1]$	$a_1$
$S * S$	$[0]$	$a_2$
$SS * S$	$[0, 1]$	$a_1$
$S * S * S$	$[0]$	$a_0$
$* S * S$	$[0]$	
$S * S$	$[0, 3]$	$a_0$
$* S$	$[0, 4]$	<b>MATCH</b>
$S$	$[0, 3]$	$a_0$
$\epsilon$	$[0, 4]$	<b>MATCH</b>

# EXAMPLE - OPTIMISATION

## ELIMINATION OF BOTH $\epsilon$ -TRANSITIONS

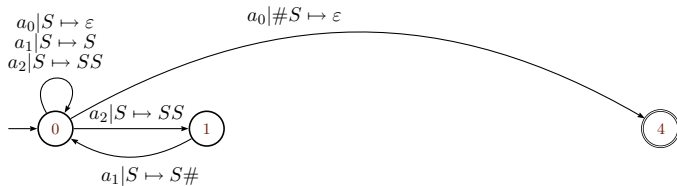
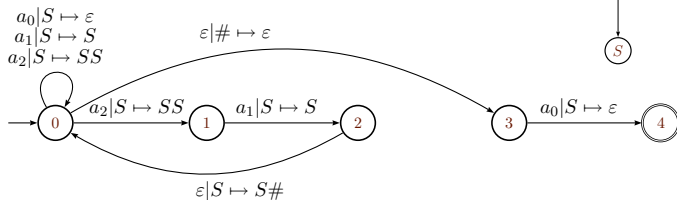
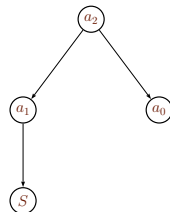
PATTERN  $P = a_2a_1Sa_0$



# EXAMPLE - OPTIMISATION

## ELIMINATION OF BOTH $\epsilon$ -TRANSITIONS

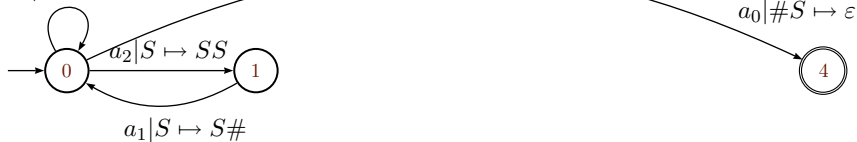
PATTERN  $P = a_2a_1Sa_0$



# EXAMPLE - OPTIMISATION

$a_0|S \mapsto \varepsilon$   
 $a_1|S \mapsto S$   
 $a_2|S \mapsto SS$

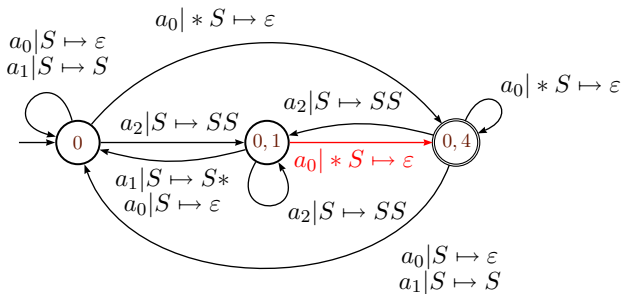
PATTERN  $P = a_2a_1Sa_0$



	$a_0$	$a_1$	$a_2$
0	$0 S \mapsto \varepsilon$ $4 \#S \mapsto \varepsilon$	$0 S \mapsto S$	$0 S \mapsto SS$ $1 S \mapsto SS$
1		$0 S \mapsto S\#$	
4			

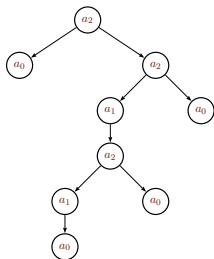
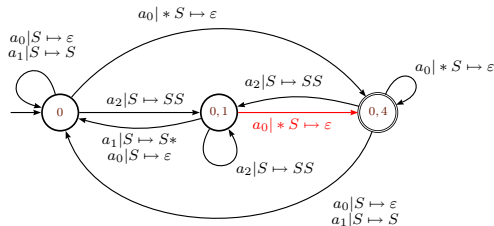
	$a_0$	$a_1$	$a_2$
[0]	$[0] S \mapsto \varepsilon$ $[0, 4]   * S \mapsto \varepsilon$	$[0] S \mapsto S$	$[0, 1] S \mapsto SS$
[0, 1]	$[0] S \mapsto \varepsilon$ $[0, 4]   * S \mapsto \varepsilon$	$[0] S \mapsto S^*$	$[0, 1] S \mapsto SS$
[0, 4]	$[0] S \mapsto \varepsilon$ $[0, 4]   * S \mapsto \varepsilon$	$[0] S \mapsto S$	$[0, 1] S \mapsto SS$

# EXAMPLE - OPTIMISATION



	$a_0$	$a_1$	$a_2$	$\epsilon$
$[0]$	$[0] S \mapsto \epsilon$ $[0,4] *S \mapsto \epsilon$	$[0] S \mapsto S$	$[0,1] S \mapsto SS$	
$[0,1]$	$[0] S \mapsto \epsilon$ $[0,4] *S \mapsto \epsilon$	$[0] S \mapsto S^*$	$[0,1] S \mapsto SS$	
$[0,4]$	$[0] S \mapsto \epsilon$ $[0,4] *S \mapsto \epsilon$	$[0] S \mapsto S$	$[0,1] S \mapsto SS$	

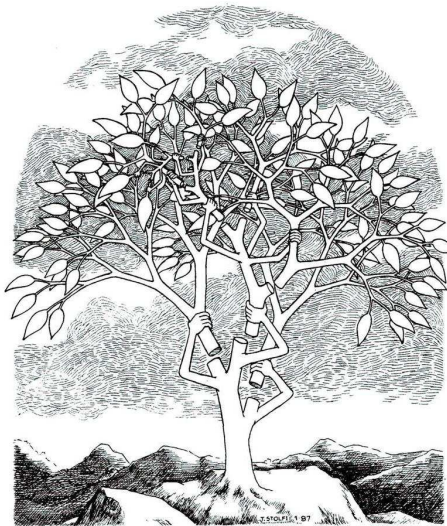
# EXAMPLE - OPTIMISATION



$P = a_2a_1Sa_0$

$T = a_2a_0a_2a_1a_2a_1a_0a_0a_0$

PDS	STATE	NODE	
$S$	$[0]$	$a_2$	
$SS$	$[0, 1]$	$a_0$	
$S$	$[0]$	$a_2$	
$SS$	$[0, 1]$	$a_1$	
$S * S$	$[0]$	$a_2$	
$SS * S$	$[0, 1]$	$a_1$	
$S * S * S$	$[0]$	$a_0$	
$*S * S$	$[0]$	$a_0$	
$*S$	$[0, 4]$	$a_0$	MATCH
$\varepsilon$	$[0, 4]$		MATCH



WEB PAGES <http://www.arbology.org>  
<http://www.arbology.com>  
COMING SOON ...

# TREE LANGUAGES, TREE AUTOMATA AND DETERMINISTIC PUSHDOWN AUTOMATA

**REGULAR TREE LANGUAGES** ARE ACCEPTED BY **FINITE TREE AUTOMATA**.

**DETERMINISTIC PUSHDOWN AUTOMATA** ACCEPT A PROPER SUPERCLASS OF THE REGULAR TREE LANGUAGES IN PREFIX OR POSTFIX NOTATION.

THIS IS PROVED IN:

JANOŠEK, J., MELICHAR, B.: *On Regular Tree Languages and Deterministic Pushdown Automata*. SUBMITTED. 2008.

THIS PAPER CONTAINS ALSO ALGORITHM OF TRANSFORMATION OF ANY FINITE TREE AUTOMATON TO AN EQUIVALENT DETERMINISTIC PUSHDOWN AUTOMATON.